Production Function

Set 4

1. What is meant by inputs or factors of production?

⟶ Inputs or factors of production refers to all those goods and services which are used in the process of production. Eg: Raw materials, labour, land etc

2. Name the types of inputs or factors of production with examples.

⟶ factors of production are classified as —

(i) Fixed factors (ii) Variable factors.

Fixed factors are those the application of which does not change with change in output. Eg: Machines, factory building etc.

Variable factors, on the other hand, are those the application of which varies with the change in output. Labour is an example of a variable factor.

3. Define production function with two features.

⟶ Production function of a firm refers to the physical relationship between inputs used and output produced by the firm under given technology. A production function is generally expressed as —

$$Q = f(x_1, x_2).$$

It says that by using x_1 units of factor 1 and x_2

of factor 2 we can almost at most produce a amount of the commodity.

The two important features of a production function are —

(i) Production function signifies the technical relationship between inputs and outputs.

(ii) It is determined by the state of ~~tech~~ technology.

4. Write a production function with one fixed factor and other variable factor.

→ When one factor is fixed and the other is a variable factor, production function may be specified as under —

$$Q_x = f(L, \bar{K})$$ where,

Where, Q_x = Output of good-x

L = Labour, a variable factor.

$\bar{K}$ = Capital, a fixed ~~factor~~ factor.

In this type of production function, output can be increased only by increasing the application of L (the variable factor).

5. Define and distinguish between short run and long run.

⟶ Short run is a time period which is less than the minimum required to bring about changes in fixed factor. On the other hand, long run is a time period in which all factors of production (fixed as well as variable) can be changed. The distinctions between the two may be given as under.

Short run	Long run
(i) Output can be increased by change in the quantity of only variable inputs like labour, raw materials etc.	(i) Output can be increased by making changes in the quantity of both fixed as well as variable inputs.
(ii) Factors of production are divided into two parts — fixed factors and variable factors.	(ii) In the long run all factors are variable.

6. Explain the concepts of total product (TP), Average Product (AP) and Marginal product (MP).

⟶ In the context of production function, three concept of product are used, viz. TP, AP and MP. To explain this concepts let us refer to the following production schedule.

Units of land ($\bar{x}_2$)	Units of labour (x_1)	TP	MP	AP
1 acre	1	2	2	2
1 "	2	5	3	2.5
1 "	3	9	4	3
1 "	4	12	3	3
1 "	5	14	2	2.8
1 "	6	15	1	2.5
1 "	7	15	0	2.1
1 "	8	14	-1	1.7

Total Product : It is the sum total of output of each unit of the variable factor used in the process of production. We write it as —

$$Q = f(x_1, \bar{x}_2)$$

Here, we have kept factor-2 (i.e. land) constant at the amount x_2 and varied factor-1 (i.e. labour). Then for each value of x_1 (labour) we get a value of Q (i.e. TP). In the above table it is seen that fixed factor is 1 acre of land and as we apply more and more units of labour (variable factor), we get different levels of total product. TP is also called TPP (Total Physical Product).

Average Product : It is defined as the output per unit of variable input. We calculate it as —

$$AP = \frac{TP}{x_1}$$

where,

where, X_1 is the level of employment of the variable input. The above table shows that at one unit of labour AP is 2 units of output. At 2 units of labour AP is 2.5 units of output. AP is also called APP (Average Physical Product)

Marginal Product : MP of an input is defined as the change in output per unit of change in the variable input when all other inputs are held constant. When factor-2 is held constant, the marginal product of factor-1 is —

$$MP_1 = \frac{\text{Change in output}}{\text{Change in input}}$$

$$= \frac{\Delta Q}{\Delta X_1}$$

Where,

Δ represents the change in variable input.

MP (also called MPP), can also be calculated in the following way —

MP_1 = TP_{x_1} units − $TP_{(x_1-1)}$ units.

7. Define and distinguish between return to a factor and return to scale.

—— When a producer effects a change in production by keeping the quantity of some factors fixed then the relationship between output and variable input is ~~refere~~ referred to as return to a factor. It may be of three types — increasing returns,

...ial return and diminishing return.

On the other hand, when a producer effects a change in production by changing all factor simultaneously then it is referred to as returns to scale. Like return to a factor, returns to scale is also of three types — increasing returns to scale, constant returns to scale and diminishing returns to scale.

Following are the points of contrast between returns to a factor and returns to scale:

(i) Returns to a factor apply when one factor alone is variable and other factors remain fixed but returns to scale apply when all factors are variable.

(ii) Returns to scale is only a long run possibility while returns to a factor are often studied with reference to short period.

8. What do you mean by an increasing & returns to a factor and diminishing return to a factor?

→ The law of increasing returns to a factor states that with a fixed amount of some factors of production, when more and more units of a variable factor are increased, total product increases at an increasing rate.

★ On the other hand the law of diminish
returns to a factor states that with a fixed
amount of some factors of production, when more
and more units of a variable factor are
increased, total product increases at a diminishing
rate.

★9. What are Constant Returns to Scale (CRS), Increasing Returns to Scale (IRS) & Diminishing Returns to Scale (DRS)?

⟶ CRS holds when a proportional increase in inputs results in an increase in output by the same proportion.

IRS holds when a proportional increase in all inputs results in an increase in output by more than the proportion.

DRS holds when a proportional increase in all inputs results in an increase in output by less than the proportion.

10. Explain the relationship between —

(a) Total product (TP) and Marginal Product (MP)

(b) ~~Marginal~~ Average Product (AP) and Marginal Product (MP)

⟶ (a) The relationship between TP and MP may be as given under :—

(i) So long as MP is increasing, TP is increasing at an increasing rate.

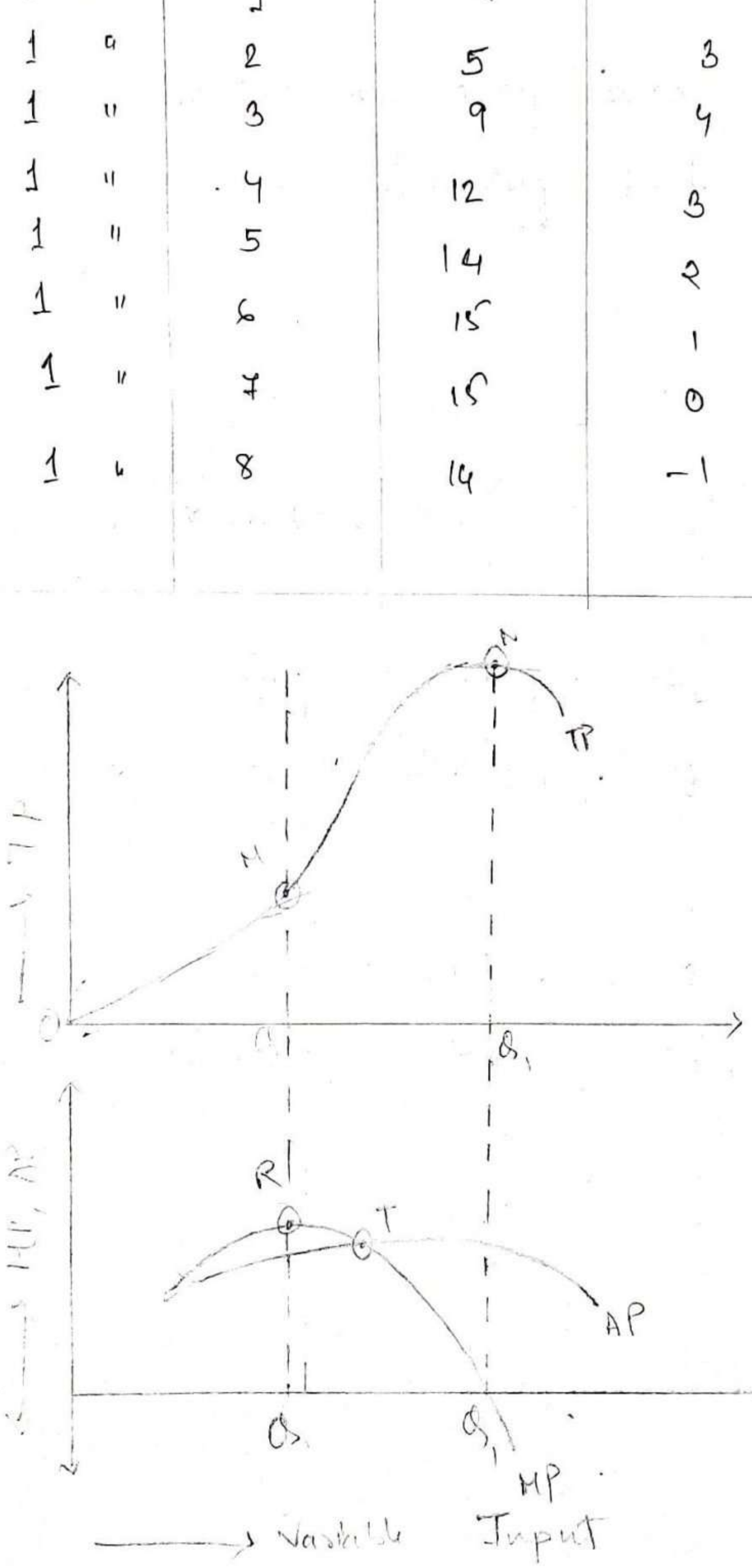
(ii) When MP starts diminishing (but positive), TP increases only at a diminishing rate.

iii. When MP = 0, there is no addition to TP.

iv. When MP is negative, TP starts declining.

The above relationship may also be explain with the help of the following table and diagram as given below :—

Units of land ($\bar{x}_2$)	Units of labour (x_1)	TP	MP	AP
1 acre	1	2	2	2
1 "	2	5	3	2.5
1 "	3	9	4	3
1 "	4	12	3	3
1 "	5	14	2	2.8
1 "	6	15	1	2.5
1 "	7	15	0	2.1
1 "	8	14	-1	1.7



b. The relationship between AP and MP are as.

i. AP increases so long as MP > AP.

ii. AP decreases when MP < AP.

iii. AP is at its maximum when AP = MP.

iv. MP may be 0 or negative but AP continues to be positive.

Q. Why MP initially rises but later on falls?

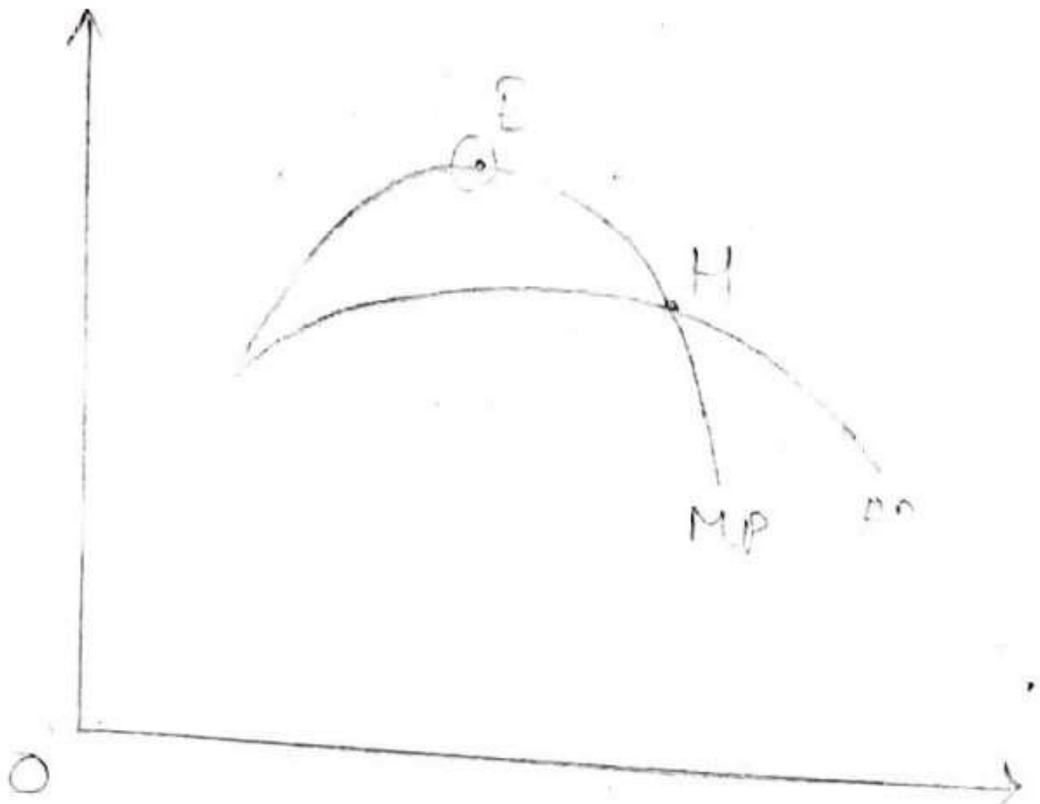
OR.

What is the reason behind ~~a~~ law of variable proportion or law of diminishing ~~r~~ return?

⟶ The reason behind the law of diminishing returns or the law of variable proportion is the following ——

As we hold one factor input fixed and keep increasing the other, the factor proportions change. Initially as we increase the amount of variable input, the factor proportions become more and more suitable for production and marginal product increases. But after a certain level of employment, the production process becomes over crowded with the variable input and the factor proportions become less and less suitable for the production. It is from this point that the marginal product of the variable input starts falling.

to rise. Once MP has fallen sufficiently, its value becomes less than the prevailing AP and the latter also starts falling. So, AP curve is also of inverse U-shaped as shown below.



13. Why does MP curve cut AP curve from above?

→ This is because, when AP rises, ~~MP~~ MP greater than AP and when AP falls, MP less than AP. It therefore follows that MP curve cuts AP curve from above, at its maximum.

14. Point out few assumptions of the law of variable proportion. Also mention two reasons / causes each for law of increasing return and law of diminishing return.

→ Assumptions of the law :—

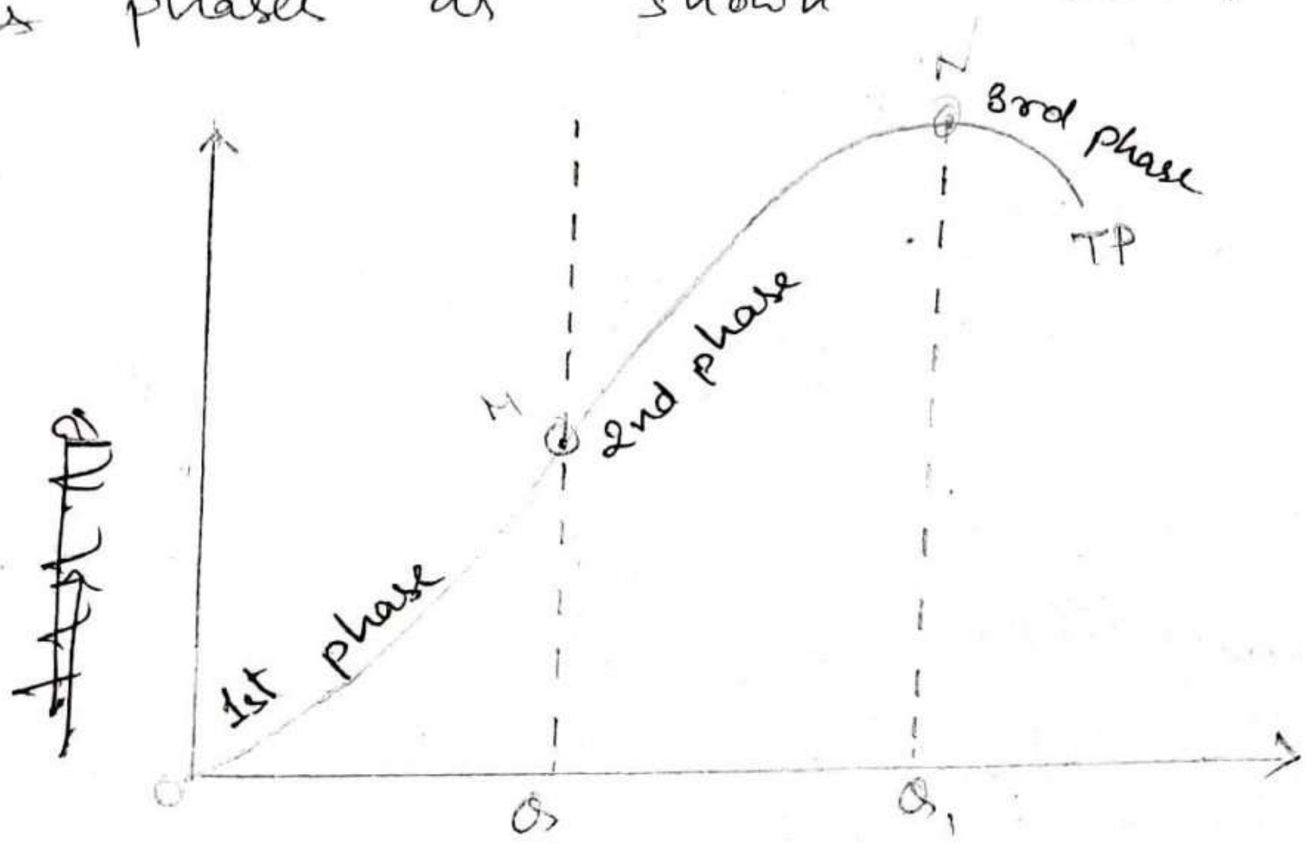
(i) The ratio in which factor proportions are combined can be changed.

(ii) The various units of the variable factor are homogenous

(iii) ~~Stra~~ State of technology remains constant.

12. Explain the shapes of TP, AP and MP curve.

⟶ Shapes of TP curve : When more and more units of a variable factor are employed with fixed factors, initially TP curve increases at an increasing rate, then increases at a diminishing rate and finally starts falling. Thus, TP curve passes through three ~~faces~~ phases as shown below.



Shapes of AP and MP curve : The marginal product of an input initially rises and then after certain level of employment it starts falling. The MP curve, therefore looks like an inverse U shaped curve. For the first unit of the variable input, MP and AP are the same. Now as we increase the amount of variable input both MP and AP rises but AP rises less than MP. Then after a point, MP starts falling. However as long as the value of MP remains higher than the latter value of prevailing AP, the ~~later~~ continues

Causes of increasing return to a factor:

i. At the ~~im~~ initial stages fixed factor (such as machine) remains under-utilized. Its fuller utilization calls for greater application of the variable factor (such as labour). Hence, initially, ~~margic~~ marginal product of the variable factor tends to increase.

ii. So long as fixed factor remains under-utilized, additional application of the variable factor tends to improve the degree of co-ordination between the fixed and variable factor. As a result, total output increases at an increasing rate.

Causes of diminishing returns to a factor:

i. As more and more units of the variable factor continues to be combined with the fixed factor the latter gets over utilized. Hence the diminishing returns.

ii. Continuous increasing application of the variable factor along with the fixed factor beyond a ~~point~~ point results in poor co-ordination between the fixed and variable factors. The law of diminishing returns accordingly sets in.

15. Explain the law of variable proportion with the help of a diagram.

→ (The law of variable proportion states that as more and more units of the variable factor is combined with the fixed factor, a state must ultimately come when marginal product of the variable factor starts declining.)

(The law is applicable particularly during the short run, when a firm can change its output by varying only the variable factor while other factors remaining unchanged. In this case the factor proportion (~~fixed fact~~ $\left(\frac{\text{Fixed factor}}{\text{variable factor}} = \frac{\bar{x}_2}{x_1}\right)$ will gradually fall with an increase in the amount of variable factor. Since $\bar{x}_2$ remains constant hence the law is known as the law of variable proportion.)

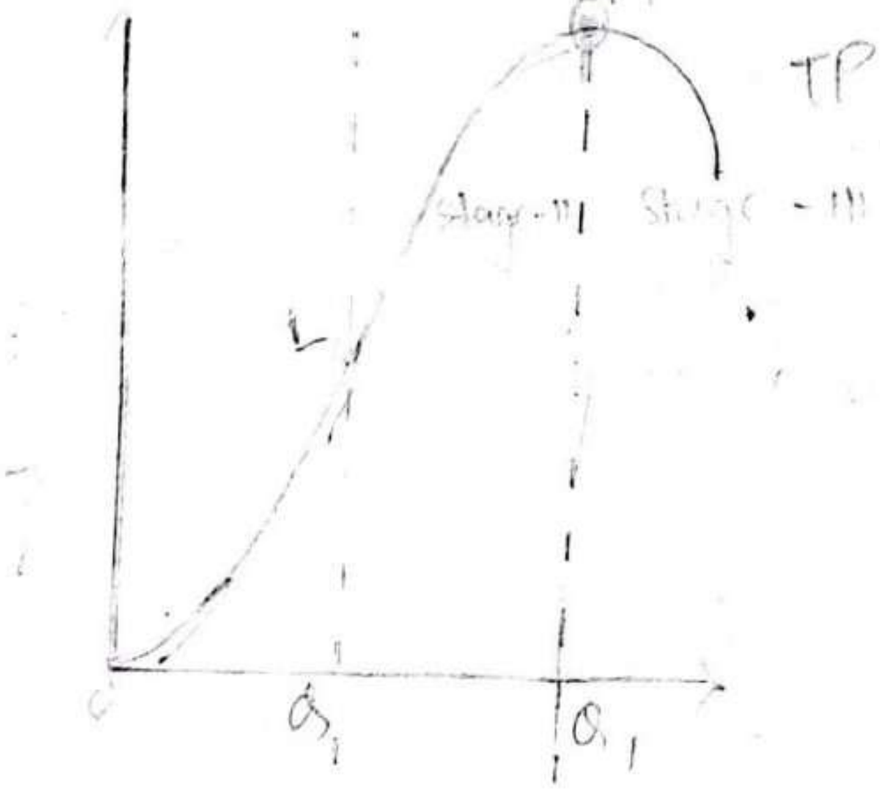
This law can be explain with the help of a schedule and diagram as given below.

Units of land ($\bar{x}_2$)	Units of labour (x_1)	TP	MP	Stages
2 acres	1	2.	2	Increasing MP
2 "	2	5.	3	
2 "	3	9	4	
2 "	4	12	3	Diminishing MP
2 "	5	14.	2	
2 "	6	15	1	
2 "	7.	15.	0	Negative MP.
2 "	8	14	-1	

Table 1: Law of variable Proportion

1 - 2
2 - 3
3 - 4
4 -> 3

The above table shows that as more and more units of labours are used, MP tends to rise till 3 units of labour are employed. In this situation, TP increases at an increasing rate. This is a situation of increasing returns to the factor. But with the application of 4 unit of labour, situation of diminishing returns sets in — MP starts decreasing and TP increases only at a diminishing rate. When MP is 0, TP reaches maximum at 15; but after that if more of labours is used, MP become negative and TP starts declining



→ Variable proportion

From the diagram it is seen that when MP is rising TP tends to rise at an increasing rate. This occurs till point I on the TP curve this corresponds to the situation of

increasing return, MP tends to decline and ~~the~~ increases only at a diminishing rate. This occurs between R and Q, on MP ~~curve~~ curve and between L and M on the TP curve. This corresponds to the situation of diminishing return to ~~be~~ a factor. However ~~exceeds~~ when employment of labours ~~exceeds~~ " OQ_1 MP becoms negative and TP starts declining.

16. Make a comparision between law of variable proportion and law of diminishing return.

→ The law of diminishing return states that if increasing quantities of a variable input are applied to a given quantity of a fixed input, the marginal product of the variable input will eventually decline. It is normally impossible to use to fixed input efficiently with only a small quantity of the variable input. In this case ~~a~~ the applications of increasing quantity of the variable input would make all the units of fixed input more productive than they were before. According to the law of diminishing return, the scope for increasing returns must eventually disappear and sooner or later the marginal product of variable factor will decline.

The law of variable proportion is some what a related concept. It says that the marginal product of a variable factor initially rises but after a certain level of employment, it starts

ing. The law of diminishing return is thus called the law of variable proportion. Infact the law of variable proportion is a ~~mordern~~ modern version of the law of diminishing return.

Cost.

1. Define cost. Write a note of different types of cost.

→ Cost / cost of production refers to the expenditure incurred by a firm on the factor inputs as well as non-factor inputs for the production of a commodity.

Different types of costs can be named as under :—

(Economic Cost = Explicit + Implicit)

(i) Explicit cost and Implicit cost :— The expenditure for hired or purchased factors of production are called explicit cost. While the cost of self owned or self supplied resources are called implicit cost.

(ii) Production cost and selling cost :— Expenditure needed to give final shape of a commodity is called production cost while expenditure needed to popularise the final good is called selling cost.

(iii) Fixed cost / supplementary cost / unavoidable cost and variable cost / prime cost / avoidable cost :— fixed cost refers to the expenditure by the producer on fixed factors of production like plant and machinery. Fixed cost remains the same whether output is 0 or maximum.

On the other hand, variable costs are the expenditure incurred by the producer on the use of variable factors of production like labour, raw materials etc. Variable costs are 0 when output is 0 but these costs increase when output increases and decreases.

(iv) AC and MC : AC is $\frac{TC}{\text{no. of units produced}}$, i.e., it is the cost per unit of output. AC is the sum of AVC and AFC.

MC on the other hand is the cost of producing an addition unit of output. In other words MC is the addition made to total cost by producing 1 more unit of output.

2. Define cost function.

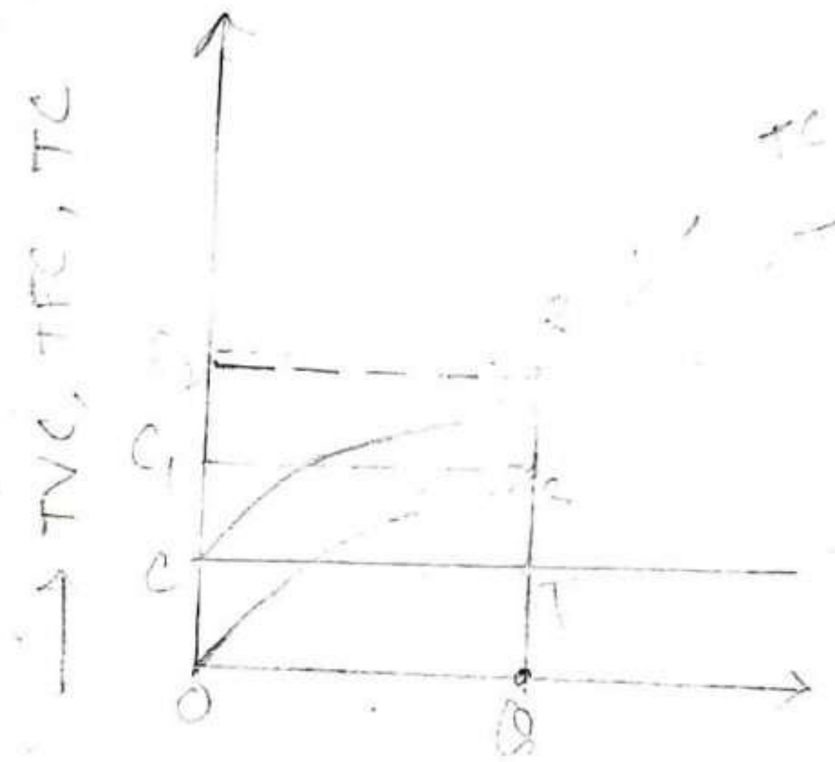
⟶ A cost function studies the functional relationship between cost and output. It shows least cost combination of inputs corresponding to different levels of output. Symbolically, $C = f(Q)$

3. Explain the shapes of Total Cost (TC), Total fixed cost (TFC) and Total variable cost (TVC)

OR

Diagramatically show the relationship among short run TC, TFC and TVC.

→ We know that in the short period in order to increase the production of output the firm needs to employ more of the variable inputs. This results in an increase in total variable cost and hence, an increase in total cost. Therefore, as output increases, both total variable cost and Total Cost increase. Total Fixed cost, however, is independent of the amount of output produced and remains constant for all levels of production.



The above figure illustrates the shapes of TFC, TVC and TC curve for a firm. TFC is a constant which takes the value of OC and does not change with the change in output. It is therefore, a horizontal straight line parallel to x-axis.

From diagram it is evident that for production of OQ amount of output, TFC = OC, TVC = OC_1 and hence TC = $OC + OC_1 = OC_2$ (i.e. TC = TFC + TVC)

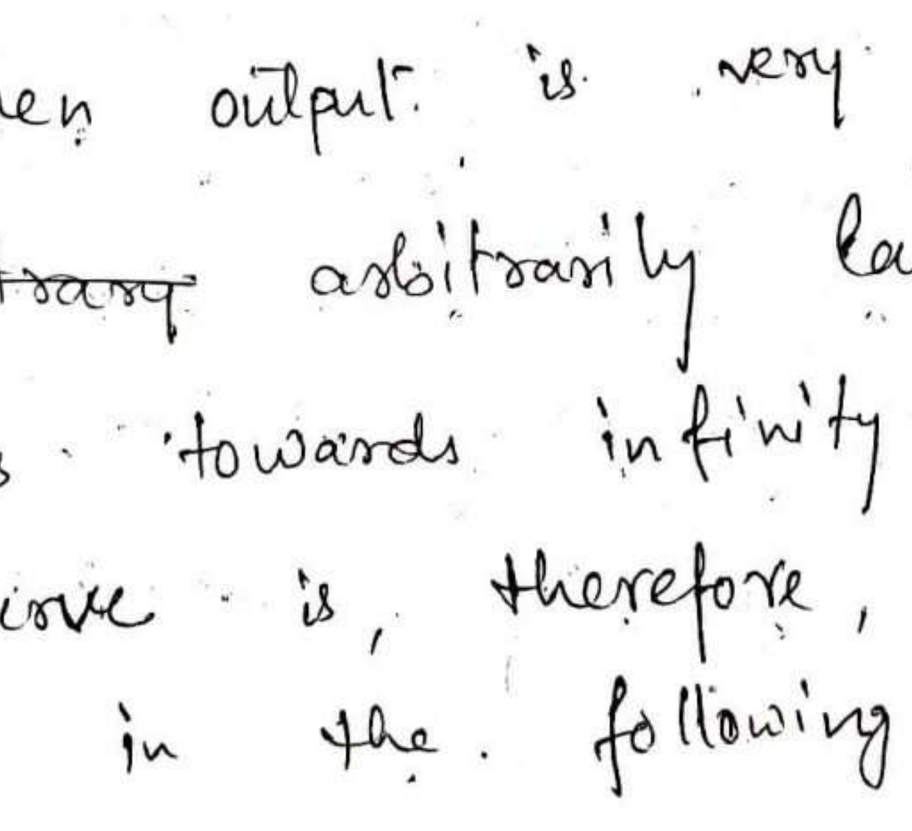
of any product.

TVC initially increases at a diminishing rate,

→ At the initial stages of production, a firm is enjoying increasing returns to a factor. It is a situation when MP of the variable factor tends to increase. Increasing MP means falling cost. When the cost of producing an additional unit is falling, TVC will be increasing only at a diminishing rate.

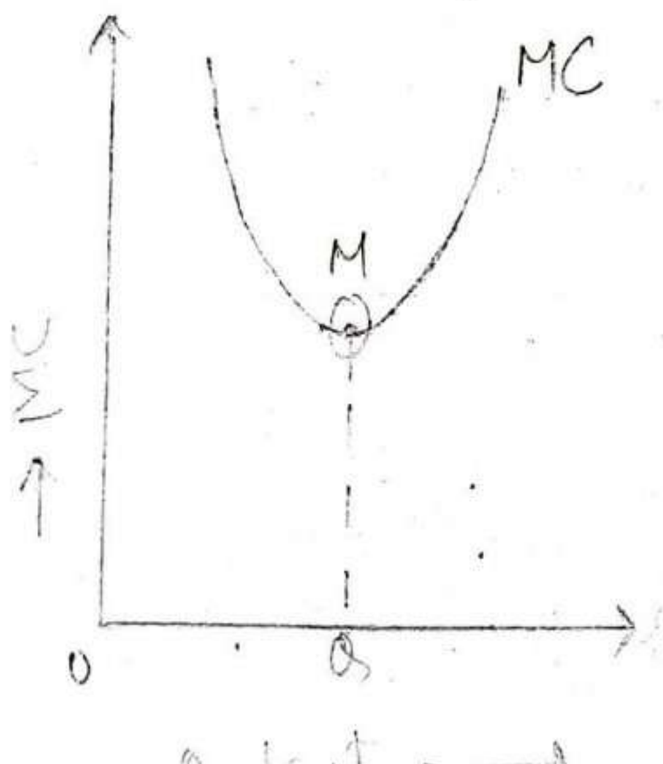
5. What is the shape of AFC curve and why does it look so?

→ AFC is obtained by dividing ~~TVC~~ TFC by total quantity produced; i.e. $AFC = \frac{TFC}{Q}$. Since TFC is constant, hence as output increase, AFC decreases. When output is very close to zero, AFC is ~~arbitrary~~ arbitrarily large and as output moves towards infinity, AFC moves towards zero. AFC curve is, therefore, a rectangular hyperbola as shown in the following diagram



6. What is the shape of Short run Marginal Cost (SMC) curve and why?

⟶ According to the law of variable proportions, initially, the marginal product of a factor increases as employment increases and then after a certain point, it decreases. This means, initially to produce every next unit of output the requirement of factors becomes less and less, and then after a certain point it becomes greater and greater. As a result with the same factor price, initially the SMC falls and then after a certain point, it rises. SMC curve is, therefore 'U-shaped' as given below—

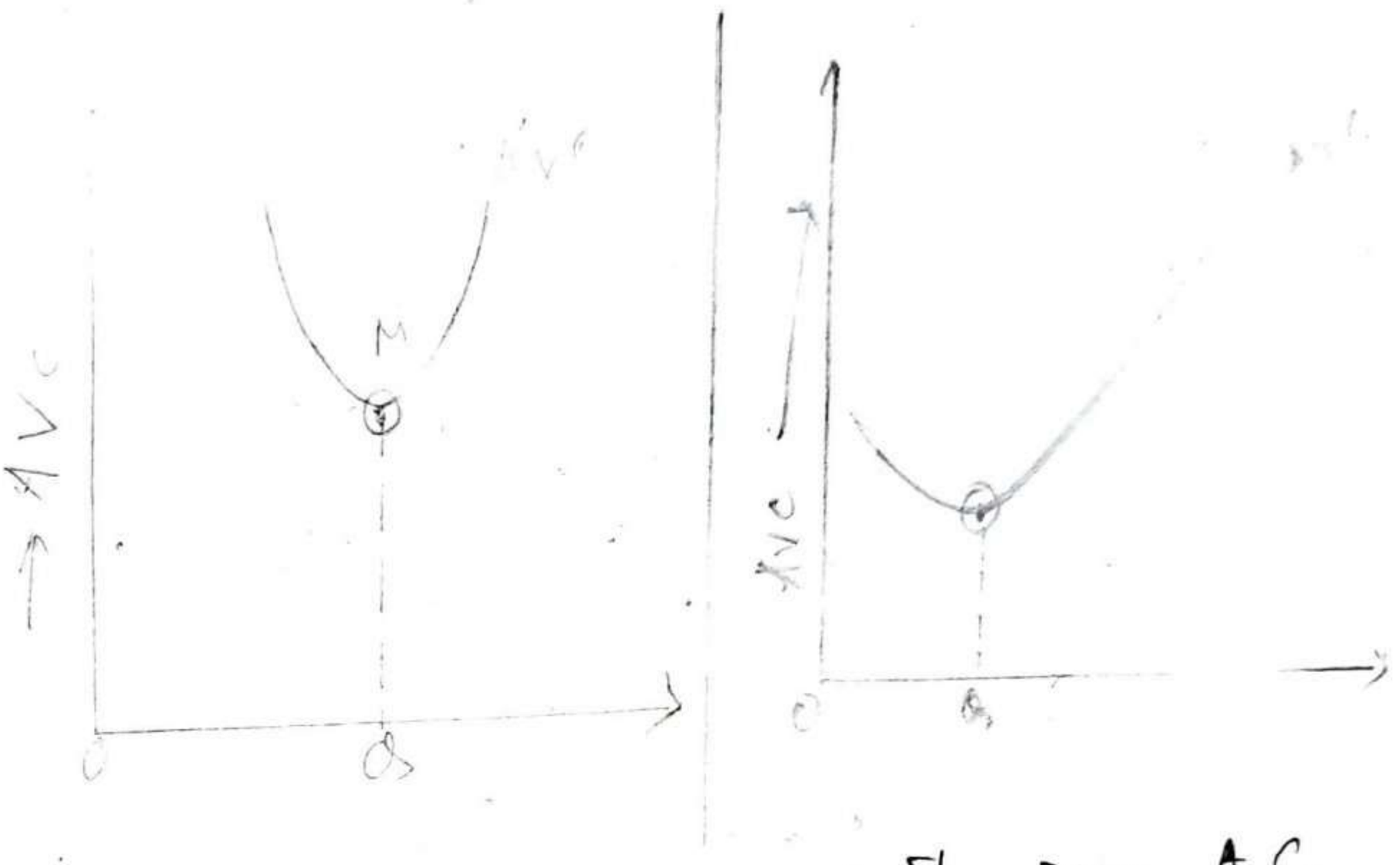


7. What is the shape of short run AVC curve and why?

⟶ AVC is obtained by dividing the total variable cost by the amount of output. That is, $AVC = \frac{TVC}{Q}$.

It should be noted that initially AVC decreases as output increases but after reaching the stage of minimum cost, AVC starts increasing

marginal is so because in the initial stages of production - the law of increasing returns to a factor operates. But after a point the law of diminishing returns begins to operate. AVC curve is therefore 'U-shaped' as shown below:



8. What is the shape of short run AC curve and why?

→ Short run AC curve is U-shaped due to the operation of law of variable proportion. This law states that initially MP of the variable factor tends to rise and eventually MP tends to fall. It is seen that as output is increased, AC first falls (due to increasing returns) reaches its minimum and then rises (due to diminishing returns). Hence AC curve also becomes U-shaped as shown below:

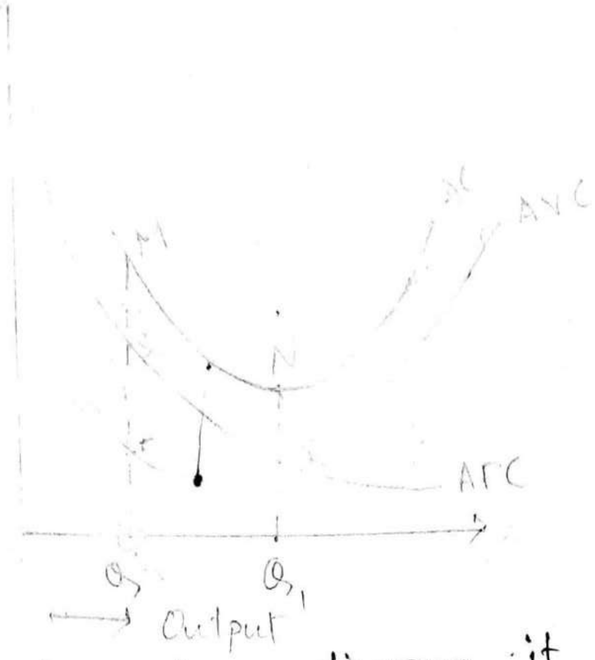
9. ~~Diagram~~ Diagramatically establish the relationship among AC, AFC and ~~ABC~~ AVC

OR

Prove that AC is the vertical summation of AFC and ~~ABC~~ AVC

→ We know that AC is the sum of ~~ABC~~ AVC and AFC. Initially both ~~ABC~~ AVC and AFC decrease as output increases. Therefore AC initially falls. But after a certain level of production, ~~ABC~~ AVC starts rising. At this stage ~~ABC~~ AVC and AFC are moving in opposite direction. Here initially, the fall in AFC is greater than the rise in ~~ABC~~ AVC and hence AC is still falling. But after a certain level of production, rise in ~~ABC~~ AVC over rides the fall in AFC. From this point onwards, AC is rising. AC curve is therefore U-shaped

The relationship among these three curves can be shown with the help of the following diagram



From the above diagram it is seen that at OQ_1 level of production. ~~AB~~ AVC = TQ_1, ~~AFC~~ AFC = SQ_1 ∴ Therefore, AC = $TQ_1 + SQ_1 = MQ_1$.

Again, at OQ_1 level of production, AVC = RQ_1, AFC = RQ_1, and hence AC = $RQ_1 + RQ_1$ = NQ_1.

10. Why when output increases, AVC tends to be closer to AC but AFC get away from AC?

⟶ AC and ~~AB~~ AVC get close to each other as output increases. On the other hand AC and AFC get away from each other as output increases. This is because as output increases, the component of AVC in AC tends to increase while the component of AFC in AC tends to decrease. However at no stage AVC will touch AC because AFC can never be

11. Why does MC imply only variable cost?

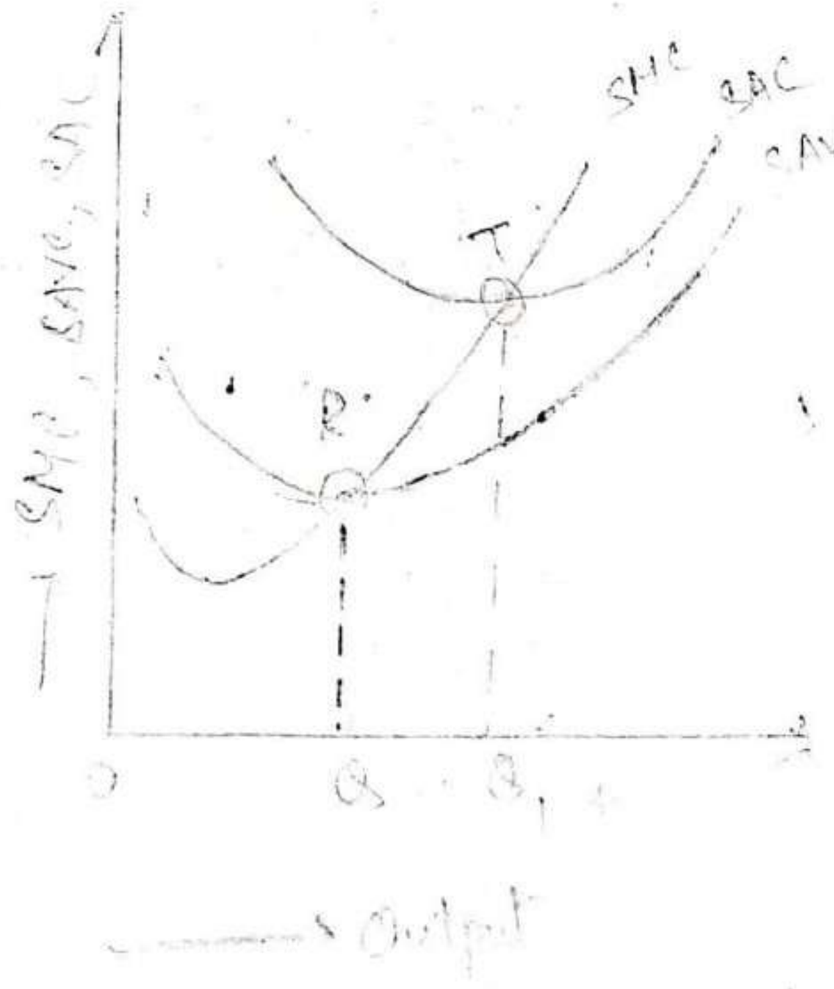
⟶ It needs to be noted that MC implies additional cost corresponding to an additional unit of output. By defination, additional cost cannot be fixed cost; it can only be variable cost. Accordingly, the sum total of marginal cost corresponding to different units of output becomes total variable cost (TVC). Thus, ~~TVC = ΣMC~~ TVC = Σ MC.

12. Explain the relationship among SMC, SAVC and SAC.

OR

Show that MC curve intersects both AVC and AC curves at their lowest points from below.

⟶ To explain the relationship let us refer to the following diagram.



It is ~~seen~~ seen from the above diagram that AVC reaches its minimum at OQ units of output. To the left of Q, AVC is falling and MC is less than AVC. But to the right of Q, AVC is rising and MC is greater than AVC. MC curve

curve at point R which is the minimum point of AVC curve. Again, it is seen from the diagram, the minimum point of AC curve is T which corresponds to the output level OQR. To the left of Q, AC is falling and MC is less than AC. Again to the right of Q, AC is rising and MC is greater than AC MC curve therefore cuts AC curve at point T which is the minimum point of AC curve.

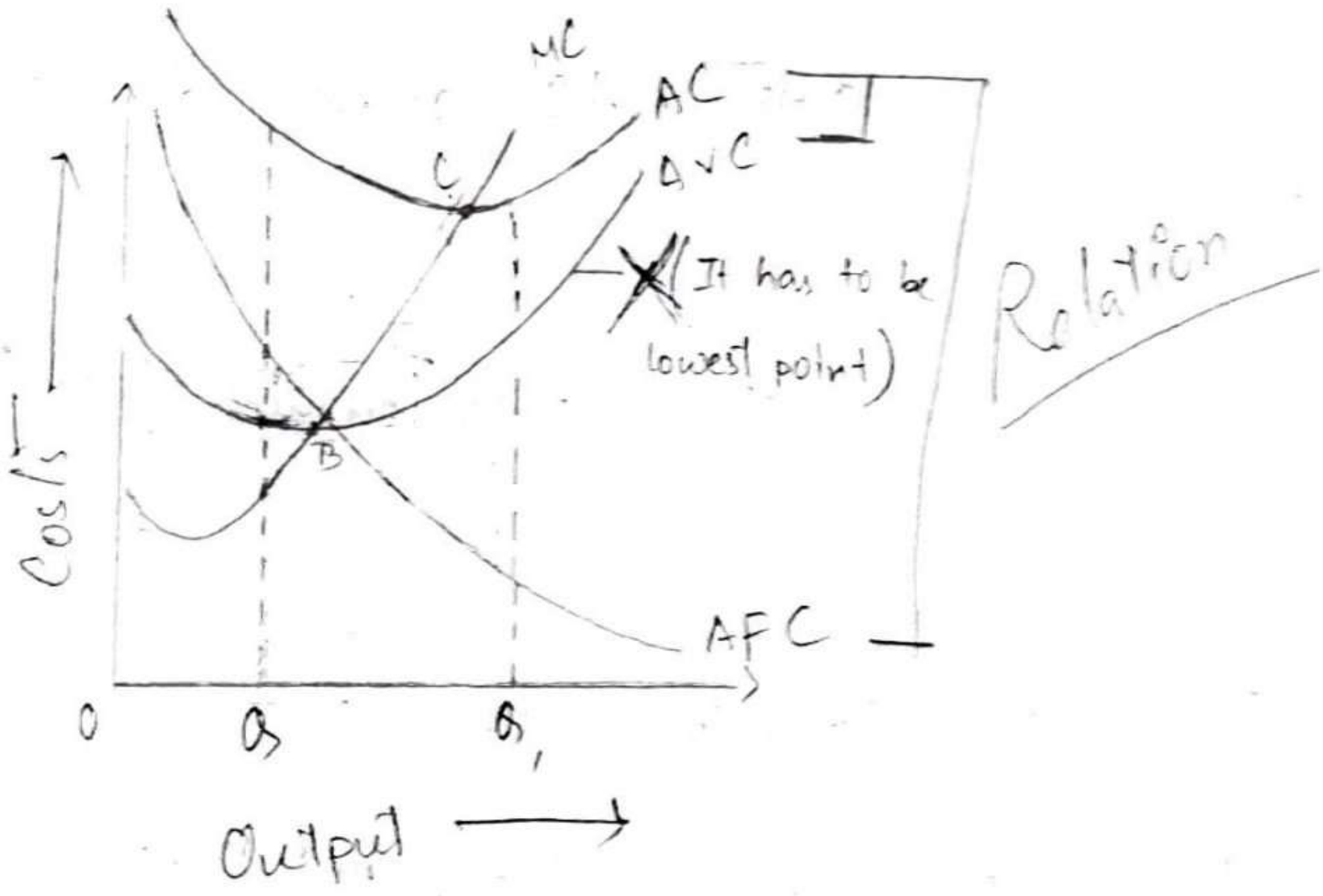
13. Why does MC curve cuts both AC and AVC at their lowest / minimum points?

→ The reason is that as long as AC and AVC are falling, MC is less than both AC and AVC. On the other hand when AC and AVC are rising, MC is higher than both. Hence MC cuts AC and AVC at their lowest points.

14. Explain the relationship among AC, AVC, and AFC and MC.

→ Initially as output increases, AVC and AC fall and then rise as output increases. The gap between AC curve and AVC curve gets smaller as output increases. On the other hand, as output increases AFC continues to fall and the gap between AC and AFC gradually increases.

It is seen from the following diagram, when output increases AC and AVC curve becomes very close to each other but never intersect. Since AFC can never be 0 or negative, AC will always be greater than AVC. MC curve like AC and AVC curves fall in the beginning then while rising, it intersects both AVC and AC at their lowest points as shown in the following figure.



Q.

15. Can MC be rising when AC falling.

→ Yes. The reason is, since MC declines faster than AC, it reaches its lowest point earlier than AC. So that MC starts rising even when AC is falling.

16. Why when AC is falling MC is less than AC?

→ It is seen that initially when output increases, MC falls more rapidly than AC and hence when AC is falling MC is less

AC.

17. Explain the relationship between AC and MC.

→ The relationships are as follows :—

i. When AC is falling, MC < AC.

ii. When AC is rising, MC > AC.

iii. When AC is constant / ~~MC~~ minimum, MC = AC

iv. MC curve cuts AC curve at its lowest point.

18. What are the relationships between AVC and MC ?

→ The relationships are —

i. When AVC is falling, MC < AVC.

ii. When AVC is rising, MC > AVC.

iii. When, AVC is constant or minimum, MC = AVC

19. What are the relationships between TC and MC

→ The relationships are —

i. When TC rises at a diminishing rate, MC decreases.

ii. When the rate of increase in TC stops diminishing, MC is at its minimum point.

iii. When TC rises at an increasing rate, MC increases.

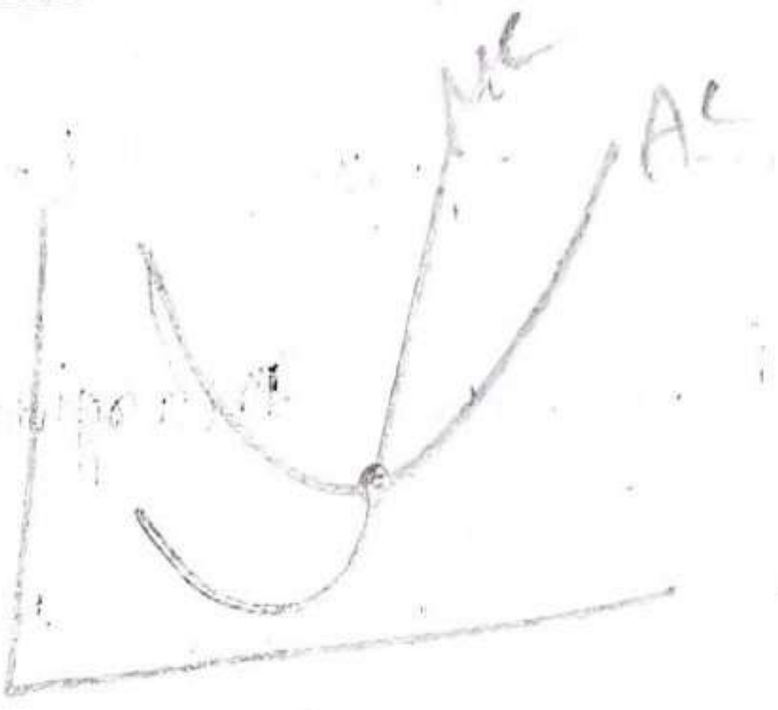
iv. MC can be calculated from TC as —

$$MC = \frac{\Delta TC}{\Delta Q}, \quad \text{or} \quad M_{n+1} = TC_n - TC_{(n-1)}$$

20. What is the relationship between AVC and AFC?

→ The relationships are :—

i.



$$AVC = \frac{TVC}{Output}$$

$$AFC = \frac{TFC}{Output}$$

30 - 20 = 10, 38 - 30 = 8, 44 - 38 = 6, 49 - 44 = 5
53 - 49 = 4, ...

Adding

Adding

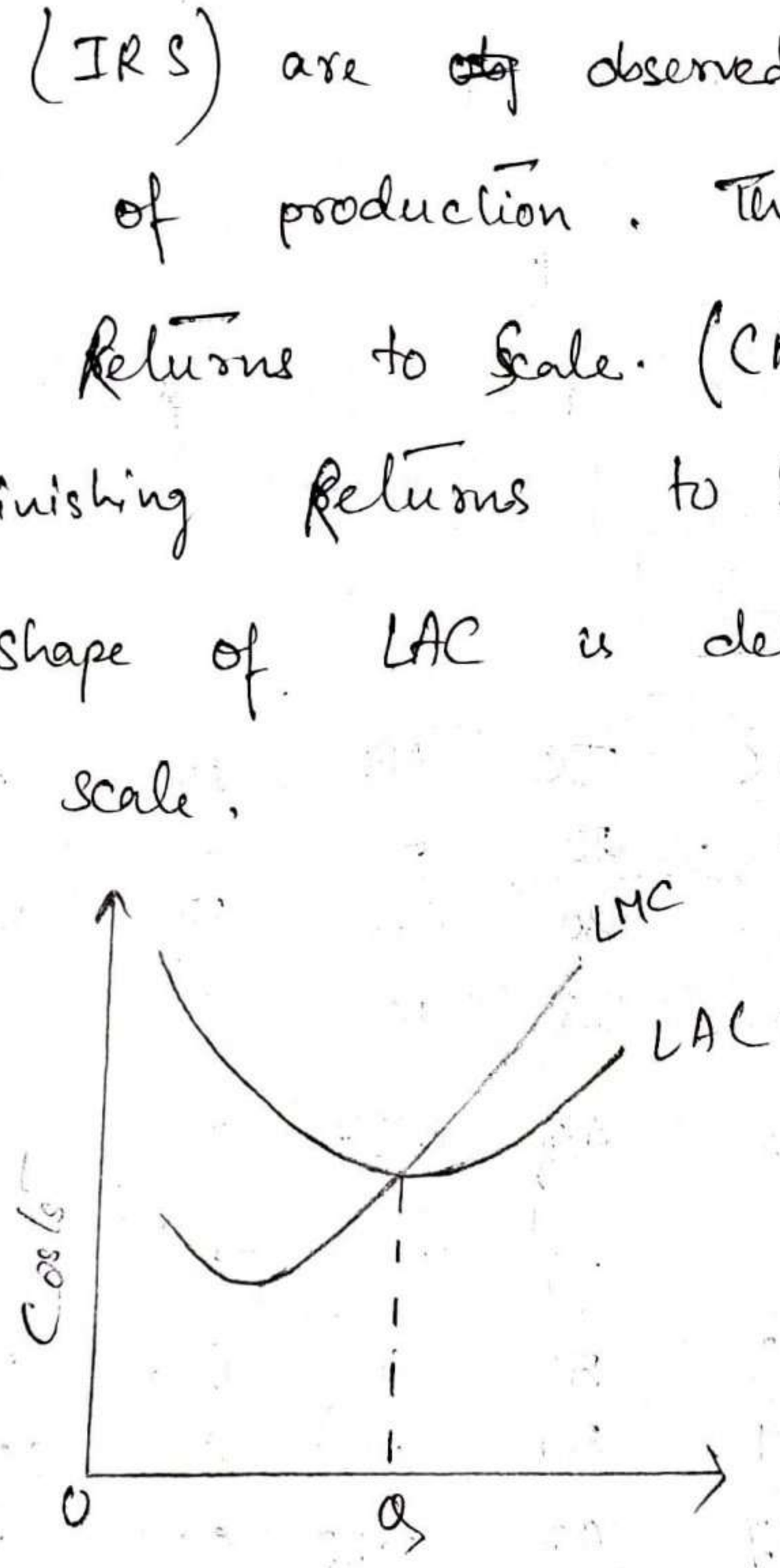
Output	TFC	TVC	TC	AFC	AVC	AC	MC
0	20	0	20	—	—	—	—
1	20	10	30	20	10	30	10
2	20	18	38	10	9	19	8
3	20	24	44	6·67	8	14·67	6
4	20	29	49	·5	7·25	12·25	5
5	20	33	53	4	6·6	10·6	4
6	20	39	59	3·33	6·5	9·83	6
7	20	47	67	2·86	6·7	9·57	8
8	20	60	80	2·5	7·5	10	13
9	20	75	95	2·22	8·33	10·55	15
10	20	95	115	2	9·5	11·5	20

Q) What are the relationships between long run cost curves?

OR

Why both LAC and ~~LAC~~ LMC curves are U-shaped?

→ In Long run, all inputs are variable and hence, there is no fixed cost like SAC and SMC curves, Longrun Average Cost (LAC) and Long run Marginal Cost (LMC) are also U-shaped. It is because that ~~increase in~~ increasing returns to scale (IRS) are ~~obs~~ observed at the initial level of production. This is then followed by Constant Returns to Scale. (CRS) and then by the Diminishing Returns to Scale (DRS) and thus the U-shape of LAC is determined by returns to ~~sc~~ scale.



→ Output.

The downward sloping portion corresponds to (Increasing Return to scale) IRS, and upward rising portion (after OQ level of output) corresponds to DRS. At the minimum point of LAC (point D shown in the fig.) CRS is observed. CRS may prevail over a range of output rather than at a single level of output. Hence, LAC will have a flat portion at the middle: