Critical Region

A set of values of the test statistic which lead to rejection of the null hypothesis is called the critical region. The Probability of with which a true null hypotheses is rejected by the test is often referred to as size of the critical region.

Let $x_1, x_2, \ldots x_n$ be a sample observation based on which the decision is to be taken about the H_0. ~~Let~~

Let Ω (omega) denotes the sample space

$\Omega = \{x_1, x_2, x_3 \ldots x_n\}$

where $x_1, x_2, x_3 \ldots$ are possible values of $X_1, X_2, \ldots X_n$.

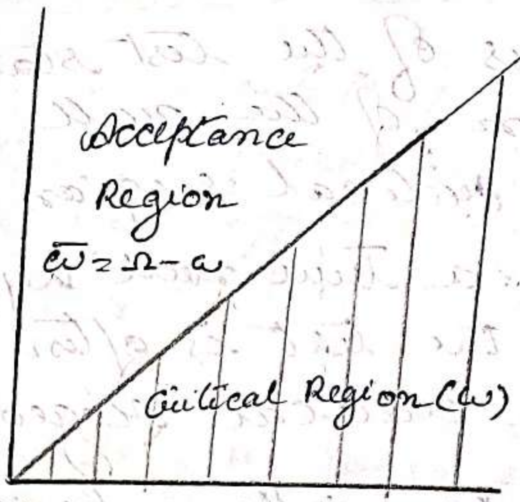
The sample space Ω can be divided into 2 of regions — ~~b) Acceptance region~~

a) critical region (ω)

b) Acceptance region ~~($\bar{\omega}$)~~

$(\bar{\omega} = \Omega - W)$

Now, the sample space is specified by the test statistic under the H_0, thus if the H_0 is rejected then the observed sample point falls in "ω" & if it falls in $\Omega - \omega$ then we accept the null hypothesis. The region 'ω' is thus called as the region of rejection of the null hypotheses & is called as the critical region which is equal to α i.e the probability (of type I error)



One Tailed & 2 tailed test

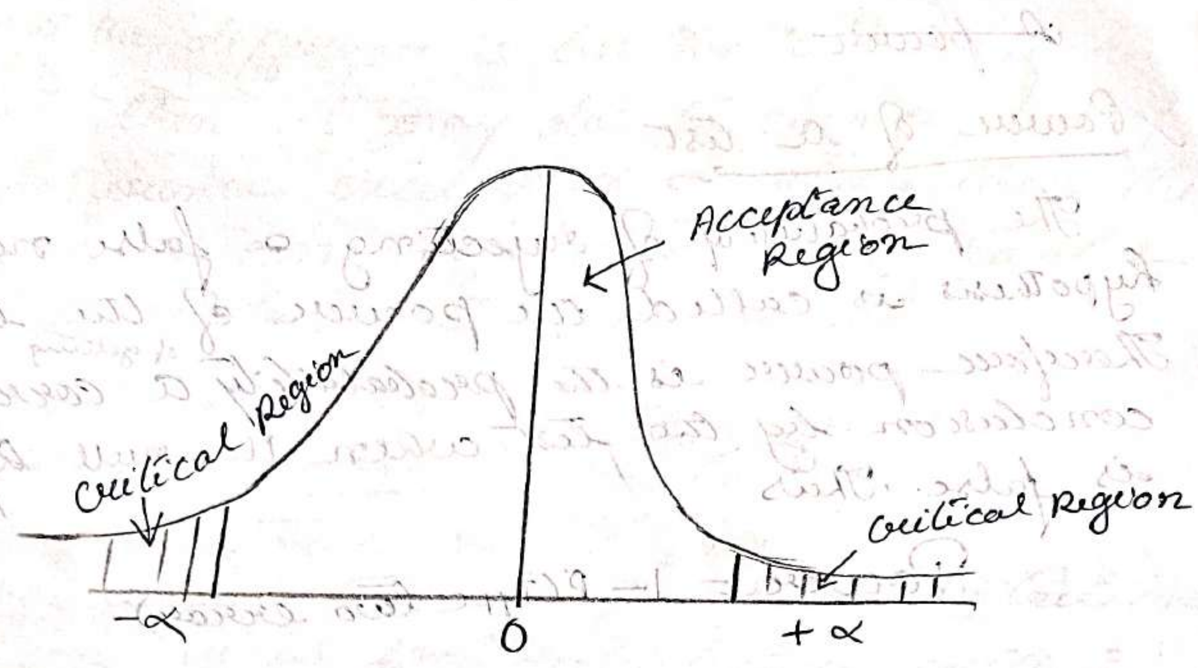
A two tail test of hypothesis will reject the null hypothesis (H_0), if the sample statistic is significantly higher than or lower than the hypothesised population parameter. Thus in a two tail test the rejection region is located in both the tails.

Let 'μ' be a population parameter & 'μ_0' be a fixed value. The null hypothesis is to be tested is $H_0 : \mu = \mu_0$

Therefore ~~Atre~~ Alternative hypothesis will be

$H_1 : \mu \neq \mu_0$.

Assuming H_0 to be true we would be looking for large difference in both sides of the expected value, i.e in both tails of the distribution, this test is therefore called as the 2 tailed test.



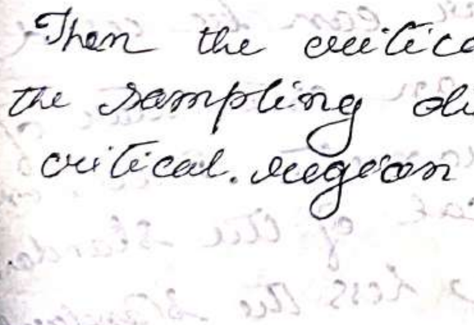
A one tailed test is so called because the rejection region will be located in only one tailed which may be either left or right depending upon the alternative hypotheses formulated. For eg- Let the null hypotheses to be tested is ~~$H_0: \mu > \mu_0$~~

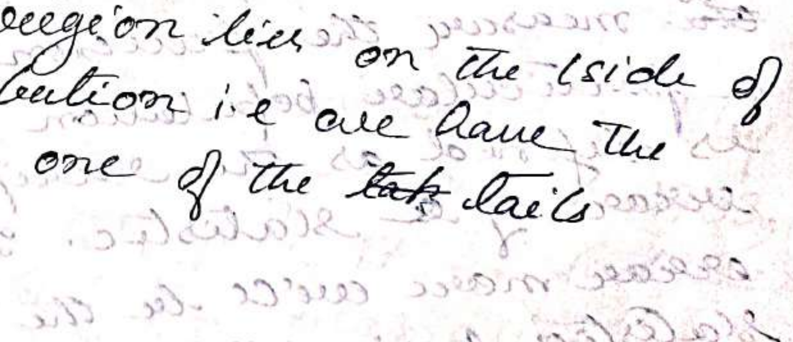
$$H_0: \mu > \mu_0$$

Then the alternative hypotheses ~~to~~ will be .

$$H_1: \mu < \mu_0$$

Then the critical region lies on the lside of the sampling distribution i.e we have the critical region in one of the ~~tals~~ tails





Power of a test

The probability of rejecting a false null hypothesis is called the power of the test. Therefore power is the probability of getting a correct conclusion by the test when the null hypothesis is false. Thus

$$\text{Power} = 1 - P(\text{Type two error})$$

Standard error

The standard deviation of sampling distribution of a statistic is known as the standard error. The standard error plays a vital role in sampling theory.

The standard error act as a vital tool for finding an interval in which the parameter is expected to fall with a certain degree of confidence. Such intervals are called as confidence interval.

The standard error can be used to measure the precision of an estimator of a particular population parameter. Precision is defined as the reciprocal of the standard error of a statistic. Thus, less the standard error more will be the precision of the statistic & vice versa.

The main purpose of the S.E is in testing whether the diff betw observed & expected values are due to chance or otherwise. The criterion usually adopted is that if the difference is less then 3 times of the S.E, then it may due to chance, but if the difference exceeds that value, then the difference is considered to be significant & cant be attributed to chance factor

Normal Distribution

Normal distribution or gaussian distribution is a continuous probability distribution with 2 parameters μ & σ and is defined by the probability density fn,

$$F(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (1)$$

where,

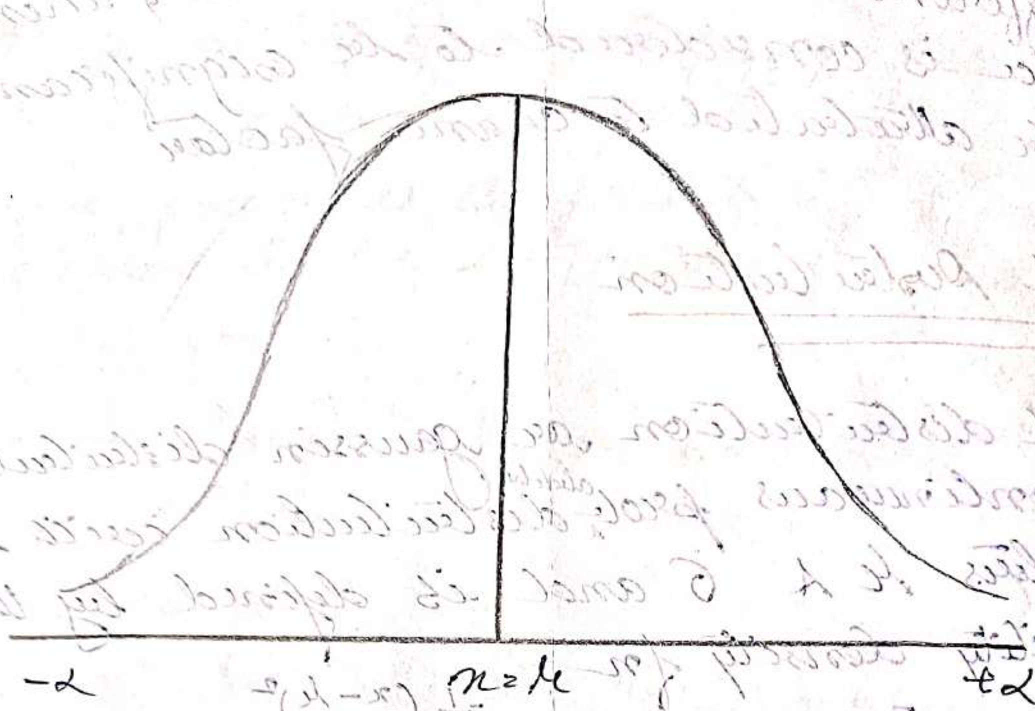
$$-\alpha < x < \alpha \ \& \ \sigma > 0$$

Here

μ is the mean & σ is the standard deviation of the distribution. π & e are the 2 mathematical constant having the approximate values as $\frac{22}{7}$ & 2.718 respectively.

Normal distribution was discovered by an english mathematician De-Moivre in 1733, used by Laplace in the year 1774, but the credit of the distribution was attributed to Gauss who 1st made the reference of the distribution in 1809.

The probability curve of normal distribution is known as the normal curve. The curve is symmetrical & bell shaped & the 2 tails extended to α in either sides as shown in the following figure.



If a random variable (x) is said normaly distributed with ~~μ~~ mean 'μ' & S. deviation σ then the random variable

$$Z = \frac{X-\mu}{\sigma}$$ is called as the 'standard normal variate' of the corresponding distribution ~~& the~~ is called the standard normal distribution. It has the density fn is

$$f(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{z^2}{2}}$$

where $-\alpha < Z < +\alpha$.

This is a special case of normal distribution with mean '0' & S.D $= \underline{1}$

Properties of normal distribution

1) Normal Distribution has 2 parameters μ & σ

2) Mean $= \mu$; ~~S.Rd~~ S. Deviation $= \sigma$

3) Mean Median & Mode are equal each being μ. Thus Mean = Median = Mode

4) The quartiles are equidistant from mean approximately $Q_1 = \mu - 0.67\sigma$

$$Q_3 = \mu + 0.67\sigma$$

Therefore Quartile deviation $= 0.67\sigma$

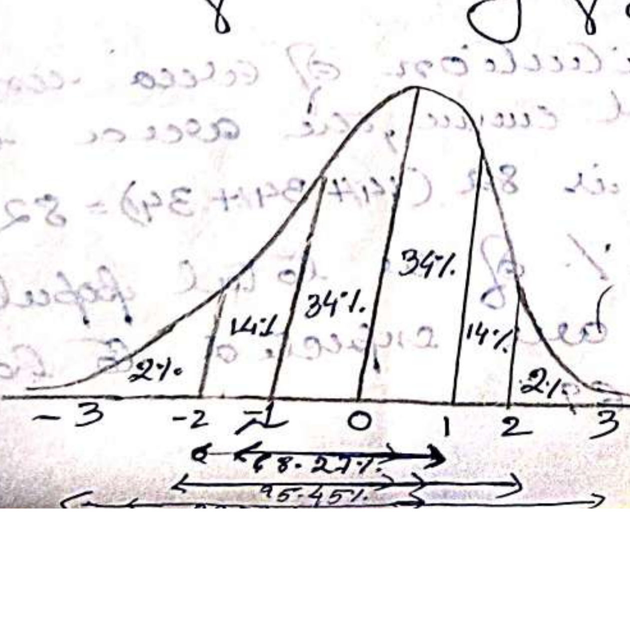
5) All odd order ~~moro~~ moments are equal to 0.

6) The coefficient of Skewness $(\gamma_1 = 0)$ & coefficient of kurtosis $(\gamma_2 = 0)$

7) The graph of the normal distribution is called the normal curve.

8) It is a bell shaped curve & is symmetrical about the mean.

8) The percentage distribution of area under the standard normal curve is ~~for~~ broadly shown in the following figure.



Area bet^{n} $Z = \pm 1$ is 68.27%.

area bet^{n} $Z = \pm 2$ is 95.45%.

$Z = \pm 3$ is 99.73%.

Importance of Normal Distribution

Normal distribution plays a very imp role in statistical theory & its application becomes usefull for the following reasons -

(1) Most of the distribution occurring in practice for ex - Poisson Distribution, Binomial Distribution etc approximated by normal distribution

2) Even if ~~the~~ a variable is not normaly distributed it can sometimes be brought to normal form by simple trasformation of variables.

3) Many of the distribution of sample statistic tend to normality & as such they can be best studied with the help of normal curve.

4) The ~~proof~~ proof of all the test of significance in sampling are based upon the fundamental assumption that population from which the sample has been drawn is normal.