PHPDSC202

LOGIC II

Unit I

Primacy of logical reasoning
Anumana : Definition, Constitution,
Processes and types.
Paksata, Paramarsa and Vyapti

1. What is Logical reasoning?

Ans: Logical reasoning is a form of thinking in which premises and relations between premises are used in a rigorous manner to infer conclusions that are entailed (or implied) by the premises and the relations. Different forms of logical reasoning are recognized in philosophy of science and artificial intelligence. *Deductive reasoning*, considered typical of mathematics, starts with premises and relations, which lead to a conclusion. For example, if A = B and B = C (the premises), the inevitable conclusion is that A = C because equality is a transitive relation. Note that if $A \neq B$ and $B \neq C$, it is not possible to draw the conclusion that $A \neq C$ because inequality is not a transitive relation. *Inductive reasoning*, necessary in empirical sciences, uses observations to arrive at premises as well as relations between premises, which are then used to arrive at conclusions. Inductive reasoning cannot convey the certainty of deductive reasoning.

2. What is the primacy of logical reasoning?

Ans: Logic has frequently been defined as the science of reasoning. That definition, although it gives a clue to the nature of logic, is not quite accurate reasoning is that special kind of thinking called inferring, in which conclusions are drawn from name is this. As thinking, however, reasoning is not the special province of logic, but part of the psychologists subject matter as well. Psychologists who examine the reasoning process find it to be extremely complex and highly emotional, Consisting of awkward trial and error procedures that are illuminated by sudden and sometimes apparently irrelevant flashes of insight. These are all of importance to psychology. Logicians, however, are not interested in the actual process of reasoning, but rather with the correctness of the completed reasoning process. There question is always does the conclusion that is reached follow from the premises used or assumed? If the premises provide adequate grounds for accepting the conclusion, if asserting the premises to be true warrants asserting the conclusion to be true also, then the reasoning is correct. Otherwise, the reasoning is incorrect. Logicians methods and techniques have been developed primarily for the purpose of making this distinction clear patients are interested in all reasoning, regardless of its subject matter, but only from this special point of view.

3. Define Anumana. What is the structure of Anumana?

Ans: Anumana takes the form of syllogism. Anu means after and mana means knowledge which comes after some other knowledge. Anumana uses knowledge derived from perception or verbal testimony and arrives at something not given by perception. Anumana or inference as defined thus, is a process of reasoning in which we pass from the perception or some mark to the knowledge of something else by virtue of the relation of universal concomitance between the two for example

The hill is fiery.

Because it smokes.

And whatever smokes is fiery.

In this example we pass from the perception of the mark smoke in the hill to the knowledge of the existence of fire in it by virtue of our previous knowledge of the universal concomitance between smoke and fire.

In every anumana there are three terms namely sadhya, paksha, hetu. These 3 terms correspond to the major term, minor term and middle term respectively of the Greek syllogism. Hetu is also called linger.

Anumana is that kind of process of reasoning which is preceded by the perception of hetu in the paksha and the remembrance of its invariable concomitance with the sadhya. So in the process of Anumana-

1. The first step is the apprehension of the hetu in the paksha

2. The second step is the Recollection of the universal concomitance between the relation of hetu and sadhya.

3.The last step is the inferential knowledge of sadhya as related to in the paksha.

From these it appears that in anumana there must be at least three propositions which are all categorical and one must be affirmative and the others may be affirmative or negative. The first proposition corresponds to the conclusion. The second proposition corresponds to the minor premise and the 3^{rd} to the major premise.

Thus we find that the order of arrangements and the propositions of anumana is the reverse of the arrangements of western syllogism.

4. What is vyapti? How is vyapti known or established?

Ans: The relation of invariable and unconditional coexistence between the middle term and the major term of an inference is technically called vyapti in Indian philosophy. For example,

Devadatta is mortal.

Devadatta is a man.

Therefore all men are mortal.

In these inference man is the middle and mortal is the major term. And the relation of invariable concomitance or coexistence between these two term is vyapti. Vyapti literally means the state of pervasion. It implies a coexistence between 2 facts someone named lee we appear which is pervaded and we are packer which is pervades.

Vyapti may be of two kinds, namely sama vyapti and ashama vyapti. A vyapti between 2 terms of equal extension is called sama vyapti. Here the vyapti holds between 2 terms which are coextensive so we may infer one of them from the other. For example, whatever is nameable is knowable and also whatever is knowable is nameable. A vyapti between two term of unequal extension, on the other hand, is called asama vyapti. The vyapti holds between 2 terms which are not coextensive. So from one we may infer the other but not vice versa. For example wherever there is smoke there is fire but whenever there is fire there may not be smoke as in the case of red hot iron ball. Now the question is how is vyapti known or how do we get a universal proposition.

According to Carvaka there is no such problem because to them inference is not a valid source of knowledge. The Buddhists base the knowledge of universal proposition on the principles of causality and essential identity. The vedantins hold that the knowledge of vyapti is the result of an unscientific induction.

The Naiyayikas agree with the vedantins but unlike them they supplement uncontradicted experience by few more step. So, according to nyaya, vyapti can be established on the basis of observation of particular instances through the knowledge of the class essence underlying such particular instances.

5. What are the different classifications of Anumana?

Ans: The Naiyayikas gives us the different classification of anumana according to their different principles according to the purpose, which an inference serves, anumana is of two kinds, namely,

1. Swarth anumana is that in which we infer to acquire knowledge for our own selves. If we infer the existence of fire in the hill and perceiving smoke in the hill remembering the universal relation between the smoke and the fire, then it is called swathanumana.

2. Parathanumana, on the other hand is that in which we infer to demonstrate the truth of the acquired knowledge before others. It is a syllogistic expression which consists of five members. They are-

A . pratijna - the heel is fiery.

B. Hetu - for it has a smoke.

C. Udaharana - wherever there is smoke there is fire as in the kitchen.

D. Anayaya - the heel has smoke.

E. Nigamana - the hill is fiery.

According to the causal relation, anumana is of three kinds, namely

1. Purvavat Anumana is that in which we infer the unperceived effect from a perceived cost. For example, when we infer future rain from the perception of dark cloud in the sky our inference is purvavat.

Two. Seshavat anumana is that in which we infer the unperceived Causeway from a perceived effect. For example, when we infer past rain from the perception of swift muddy current of the river, our inference is seshavat.

3. Samanyatadrsta anumana is that which is based on certain observed points of general similarity between the objects of experience. For example when a man infers cloven hoof of an unknown animal simply by seeing it's horn, we have an example of samanyatadrsta anumana.

According to the application of the method anumana is of three kinds, namely

1. Kavalanvayi anumana is that in which the middle term is positively related to the major term. This means that there is agreement between the two term in respect of prisons don't stop for example,

All smoky objects are fiery.

The heel is smokey.

Therefore the heel is fiery.

Here, the middle term Smokey is positively related to the major term fiery.

2. Kevalavyatireki anumana is that in which the middle term is negatively related to the major term this means that there is agreement between the two terms in respect of absence. For example,

No non fiery object Is Smokey.

The hill is smoky.

Therefore the hill is fiery.

Here, the middle term Smokey is negatively related with the major term non fiery.

3. Anvayi-vyatireki anumana is that in which the middle term is related to the major term both positively and negatively. For example

1. All smoky objects are fiery.

The hill is smoky.

Therefore the hill is fiery.

2. No non fiery object is smokey.

The heel is smokey.

Therefore the heel is fiery.

In this example there is double agreement between middle and major term. They agree in presence in the positive instance and they also agree in absence in the negative instance.

6. What do you mean by paksata?

Ans: In Sanskrit dictionary, Pakshata means 1) Alliance, partisanship. 2) Adherence to a party.

7. What is the paksha in Indian logic?

Ans: According to the nalysis in Indian logic (paksha, "the hill") "possesses" the middle term (hetu, "smoke"), which is recognized as "pervaded by" the major (sadhya, "fire"). The relation of invariable connection, or "pervasion," between the middle (smoke) and the major (fire)—"Wherever there is smoke, there is fire"—is called vyapti.

8. What is a Paksa in Nyaya philosophy?

Ans: Paksa is one of the three terms in constituents of Inference in Anumana (Inference- Nyaya). The word indicates after knowledge and it follows perception. It is defined as a specific form of knowledge preceded by perception.

9. What is Linga Paramarsha?

Ans: Paramarsha or Linga-Paramarsha or reference. Inference begins from observing the presence of proof or Hetu in the sadhya or Pakshya where it has been intimated by invariable correlation or Vyapti with the paksa or Sadhya.

10. Write a note on paramarsa.

Ans: Paramarsa can be paraphrased as 'the confirmatory knowledge'. It is a stage in the process of Inference. Inference is one of the four processes of knowing or internalizing the universe, accepted by the Indian logicians Naiyayikas). The other three processes are perceptual, analogical and verbal processes.

Although other systems of Indian philosophy accepted different numbers of the process of internalization, the Naiyayikas included all extra processes in one or the other of these four Pramanas. Carvaka accepted only one namely, perception. The Vaisesikas and the Buddhists accepted only two namely perception and Inference. The Sankhya philosophers accepted three adding verbal testimony to the above List of two. Naiyayikas accepted four by adding upamana to the above list. The Prabhakara school of Purva- mimamsa added presumption making the number five. The Bhatta school of Purva—mimamsa made it six by adding the process of knowing absence. The folk tradition added two more to the list namely history and inclusion.

The Naiyayikas included arthapatti and sambhava in inference aitihya in sabda and anupalabdhi was felt to be unnecessary since absence could be known by perception also. By 10th century A.D. the Vaisesikas and the Naiyayikas came very close in their world view and the system was more or less known as Nyaya-Vaisesika system of though. Jayanta Bhatta has clearly stated that Vaisesikastu asmad-anuyayinah eva. (Nyayamanjari Ist ahnika). The Nyaya-Vaisesika system of thought accepted five member process of anumana to convince others. The bhatta school accepted only three members. The five members are:

1. Pratijna : The Mountain possesses fire
2. Hetu : Because there is smoke
3. Udaharana : Like the Kitchen
4. Upanaya: Such is this mountain
5. Nigamana : Therefore Mountain Process fire

The Bhatta school thinks either the first three steps or the last three steps are sufficient to explain the generation of inferential cognition. This issue is being debated for more than 1500 years. Sasadhara the pre-gangesa neo-logician wrote an independent essay on Paramarsa in his Nyayasiddhanta dipa and defended the Nyaya-Vaisesika position taking into account all the pros and cons of the objection raised by the Bhatta-Mimamasakas.

Unit II

Hetvabhasa: Definition and types
Ashudha, Badhita, Satpratioaksa, Virudha, Svabhyabhichara

1. What is hetvabhasa?

Ans: The fallacies of inference (hetvābhāsa) in Indian logic are all material fallacies. So far as the logical form of inference is concerned, it is the same for all inferences. There Is, strictly speaking, no fallacious form of inference, in logic since all inferences must be put in one or other of the valid forms. Hence if there is any fallacy of inference, that must be due to the material conditions on which the truth of the constituent premises depends. It may be observed here that in the Aristotelian classification of fallacies into those in dictione and those extra dictionem there is no mention of the formal fallacies of inference like the undistributed middle, the illicit process of the major or minor term, and so forth. The reason for this, as Faton rightly points out, is that "to one trained in the arts of syllogistic reasoning, they are not sufficiently persuasive to find a place even among sham arguments." As for Aristotle's fallacies in dictione, i.e., those that occur through the ambiguous use of words, they are all included by the Naiyayika among the fallacies of chala, jāti and nigrahasthāna with their numerous subdivisions.

In Indian Logic, a material fallacy is technically called hetvābhāsa, a word which literally means a hetu or reason which appears as, but really is not, a valid reason. The material fallacies being ultimately due to such fallacious reasons, the Naiyāyikas consider all these as being cases of hetvābhāsa. According to the Naiyayi- 5 kas, there are five kinds of material fallacies. Taese are (1) Savyabhicara Satpratipaksa, (4) Asiddha, (5) Badhita.

2. Mention the different types of fallacies in Indian logic according to Nyaya.

Ans: The first kind of fallacy is called savyabhicara or the irre- gular middle. To illustrate:

All bipeds are rational;

Swans are bipeds

Therefore swans are rational.

The conclusion of this inference is false. But why? Because the middle term 'biped' is not uniformly related to the major 'rational' It is related to both rational and non-rational creatures. Such a middle term is called savyabhicara or the irregular middle.

The savyabhicara hetu or the irregular middle is found to lead to no one single conclusion, but to different opposite couclu- sions. This fallacy occurs when the ostensible middle term violates the general rule of inference, namely, that it must be universally related to the major term, or that the major term must be present in all cases in which the middle is present. The savyabhicara middle, however, is not uniformly concomitant with the major term. It is related to both the existence and the non-existence of the major term, and is, therefore, also called anaikāntika or an inconstant concomitant of the major term. Hence from such a middle term we can infer both the existence and the non-existence of the major term. To take another illustration:

All knowable objects are fiery;

The hill is knowable;

Therefore the hill is fiery;

Here the middle 'knowable' is indifferently related to both fiery objects like the kitchen, and fireless objects like the lake. All knowables being thus not fiery, we cannot argue that a hill is fiery because it is knowable. Rather, it is as much true to say that, for the same reason, the hill is fireless. " The second kind of fallacy is called viruddha or the contra dictory middle. Take this inference: "Air - is heavy, because it is empty." In this inference the middle term 'empty." is con- tradictory because it disproves the heavi ness of air. Thus the viruddha or the contradictory middle is one which disproves the very proposition which it is meant to prove. This happens when the ostensible middle term, instead of proving the existence of the major, in the minor, which is intended by it, proves its non-existence therein. Thus to take the Naiyayikas' illustration, if one argues. "Sound is eternal, because it is caused," we have a fallacy of the viruddha or contradictory middle. The middle term, 'caused does not prove the eternality of sound, but its non-eternality, because whatever is caused is non-eternal. The distinction between the savyabhicāra and the viruddha is that while the former only fails to prove the conclusion, the latter disproves it or proves the contradictory proposition.

The third kind of fallacy is called satpratipaksa or the infer entially contradicted middle. This fallacy arises when the ostensible middle term of an inference is validly contradicted by other middle term which proves the non-existence of the major term of the first inference. Thus the inference "sound is eternal, because it is audible" is validly contradicted by another inference like this: "sound is non-eternal, because it is produced like a pot. Here the non-existence of eternality (which is the major term of the first inference) is proved by the second inference with its middle term 'produced' as against the first inference with its middle 'audible.' The distinction between the viruddha and the satpratipakasa is that, while in the former the middle itself proves the contradictory of its conclusion, in the latter the con tradictory of the conclusion is proved by another inference based on another middle term.

The fourth kind of fallacy is called asiddha or sādhyasama, i. e. the unproved middle. The sadhya-sama middle is one which is not yet proved, but requires to be proved, like the that the sadhyasama middle is not a proved or an established fact, but an asiddha or unproved assumption. The fallacy of the asiddha occurs when the middle term is wrongly assumed in any of the premises, and so cannot be taken to prove the truth of the conclusion. Thus when one argues, "the sky-lotuss is fragrant because it has lotusness in it like a natural lotus," the middle has no locus standi, since the sky-lotus is non- existent, and is, therefore, asiddha or a merely assumed but not proved fact.

The last kind of fallacy is called bädhita or the non-infer entially contradicted middle. It is the ostensible middle term of an inference, the non-existence of whose major is ascer- tained by means of some other promāna or source of knowledge. This is illustrated - "Fire is cold because it is a substance". Here 'coldness' is the sadhya or major term, and 'substance' is the middle term. Now the non-existence of coldness, may more, the existence of hotness is perceived in fire by our sense of touch. So we are to reject the middle 'substance' as a con- tradicted middle. The fallacy of satpratipaksa, as explained before, is different from this fallacy of badhita, because in the former one inference is contradicted by another inference, while in the latter an infe ence is contradicted by perception or some other non-inferemial source of knowledge. Another example of badhita would be: Sugar is sour, because it produces acidity.

Unit III

Formal proof of validity: Nineteen Rules – (Direct, Indirect and Conditional Proof construction)

1. What do you mean by formal proof of validity?

Answer- A formal proof of validity for a given argument is defined to be a sequence of statements, each of which is either a premise or that argument or follows from preceding statements by an elementary valid argument and such that the last statement in the sequence is the conclusion of the argument whose validity is being proved. We first define an elementary valid argument as any argument is a substitution instance of an elementary valid argument form.

2. What is the formal way of writing out the proof or validity?

Answer- A more formal and more concise way of writing out this proof of validity is to list the premises and the statement deduced from them in one column with justification for the letter written beside them each case the justification for a statement specifies the preceding statements from which the rule of inference by which the statement in question was deduced this convenient to put the conclusion to the right of the last premises separated from it by a slanting line which automatically marks about it to be premises.

3. Illustrate an elementary valid argument.

Answer - Any substitution instance of an elementary valid argument form is an elementary valid argument thus, the argument

$\sim C \supset (D \supset E)$

$\sim C$

$\therefore D \supset E$

is an elementary valid argument because it is a substitution instance of the elementary valid argument form Modus Ponens (MP). It results from

$p \supset q$

p

$\therefore q$

But substituting '$\sim C$'for p & '$D \supset E$'for q therefore it is of that form even though Modus Ponens is not the specific form of the given argument.

4. Why are the 9 rules of inference called elementary valid argument forms?

Answer- The nine rules of inference are elementary valid argument forms whose validity is easily established by two tables they can be used to construct formal proofs of validity for a wide range of more complicated arguments. The names listed are standard for the most part and the use of their abbreviations permit formal proofs to be set down with a minimum of writing.

5. Why do we use the formal truth of validity?

Answer- If argument contains more than two or three different simple statements or components, it is tedious to use through tables to test their validity. More convenient method of establishing the validity of some argument is to deduce their conclusion from their premises by a sequence of shorter, more elementary of arguments that are already known to be valid. Consider, for example, the following arguments form in which there are five simple statements

A v (B ⊃ D)

`C ⊃ (D ⊃ E)

A ⊃ C

~C

∴ B ⊃ E

To establish the validity of this argument by means of a truth table would require a table with 32 rows. We can prove the valid argument valid however by a sequence of just four arguments whose validity has clearly been established by using the rules of inference

1. A v (B ⊃ D)

2. ~C ⊃ (D ⊃ E)

3. A ⊃ C

4. ~C / ∴ B ⊃ E

5. ~A 3,4 MT

6. B ⊃ D 1,5 DS

7. D ⊃ E 2, 4 MP

8. B ⊃ E 6, 7 HS

Rules of inference-

1. Modus Ponens (MP)

p ⊃ q

p

∴ q

2. Modus Tolens (MT)

p ⊃ q

~ q

∴ ~p

3. Hypothetical syllogism (HS)

p ⊃ q

q ⊃ r

∴ p ⊃ r

4. Disjunctive syllogism (DS)

p v q

~p

∴ q

5. Constructive Dilemma (CD)

(p ⊃ q) . (r ⊃ s)

p v r

∴ q v s

6. Absorption (ABS)

P ⊃ q

∴ p ⊃ (p.q)

7. Simplification (simp)

p.q

∴ p

8. Conjunction (conj)

p

q

∴ p.q

9. Addition (Add)

p

∴ p v q

Rule of Replacement: Any of the following logically equivalent expressions can replace each other wherever they occur:

10. De Morgan's Theorems (De M.):

$\sim(p.q) \equiv (\sim p \vee \sim q)$

$\sim(p \vee q) \equiv (\sim p. \sim q)$

11. Commutation (Com.):

$(p \vee q) \equiv (q \vee p)$

$(p.q) \equiv (q.p)$

12. Association (Assoc.):

$[p\vee(q\vee r)] \equiv [(p \vee q)\vee r]$

$[p.(q.r)] \equiv [(p.q). r]$

13. Distribution (Dist.):

$[p.(q\vee r)] \equiv [(p.q) \vee(p.r)]$

$[p\vee(q.r)] \equiv [(p \vee q).(p \vee r)]$

14. Double Negation (D.N.):

$p \equiv \sim\sim p$

15. Transposition (Trans.):

$(p \supset q) \equiv (\sim q \supset \sim p)$

16. Material Implication (Impl.):

$(p \supset q) \equiv (\sim p\vee q)$

17. Material Equivalence (Equiv.):

$(p \equiv q) \equiv [(p \supset q).(q \supset p)]$

$(p \equiv q) \equiv [(p. q) \vee (\sim p. \sim q)]$

18. Exportation (Exp.):

$[(p.q) \supset r] \equiv [p \supset (q \supset r)]$

19. Tautology (Taut.):

$p \equiv (p \vee p)$

$p \equiv (p.p)$

Q. What is conditional proof?

Ans: To every argument there corresponds a conditional statement whose ante. cedent is the conjunction of the argument's premisses and whose consequent is the argument's conclusion. As has been remarked, an argument is valid if and only if its corresponding conditional is a tautology. If an argument has a conditional statement for its conclusion, which we may symbolize as AC then if we symbolize the conjunction of its premisses as P_{1} ^ 2 the argument is valid if and only if the conditional

$P \supset (A \supset C)$. (1)

is a tautology. If we can deduce the conclusion $A \supset C$ by a sequence of elementary valid arguments, from the premisses conjoined in P, we thereby prove the argument to be valid and the associated conditional (1) to be a tautology. By the principle of Exportation, (1) is logically equivalent to

$(P.A) \supset C$. (2)

But (2) is the conditional associated with a somewhat different argument. This second argument has, as its premisses, all of the premisses of the first argument plus an additional premiss that is the antecedent of the conclusion of the first argument. The conclusion of the second argument is the consequent of the conclusion of the first argument. Now, if we deduce the conclusion of the second argument, C, from the premisses conjoined in PA by a sequence of elementary valid arguments, we thereby prove that its associated conditional statement (2) is a tautology. But since (1) and (2) are logically equivalent, this fact proves that (1) is a tautology also, from which it follows that the original argument with one less premiss and the conditional conclusion A supset C -is valid also. Now, the rule of Conditional Proof permits us to infer the validity of any argument

P

$\therefore A \supset C$

from a formal proof of validity for the argument

P

A

$\therefore C$

Given any argument whose conclusion is a conditional statement, a proof of its validity, using the rule of Conditional Proof-that is, a conditional proof of its yalidity is constructed by assuming the antecedent of its conclusion as an additional premiss, and then deducing the consequent of its conclusion by a sequence of elementary valid arguments. Thus a conditional proof of validity for the argument

$(A \vee B) \supset (C.D)$

$(D \vee E) \supset F$

∴A ⊃ F

may be written as

1. (A v B) ⊃ (CD)

2. (DvE) ⊃ F /∴A ⊃ F

3. A/∴F (C.P.)

4. A v B 3, Add.

5. C.D 1, 4, M.P.

6. D.C 5, Com.

7. D 6, Simp.

8. DvE 7, Add.

9. F 2, 8, M.P.

Q. What is indirect proof?

Ans: The method of indirect proof, often called the method of proof by reductio ad absurdum, is familiar to all who have studied elementary geometry. In deriving his theorems, Euclid often begins by assuming the opposite of what he wants to prove. If that assumption leads to a contradiction, or 'reduces to an absurdity', then that assumption must be false, and so its negation-the theorem to be proved-must be true.

An indirect proof of validity for a given argument is constructed by assum- ing, as an additional premiss, the negation of its conclusion, and then deriving an explicit contradiction from the augmented set of premisses. Thus an indirect proof of validity for the argument

A⊃ (B-C)

(BvD) ⊃E

DvA

∴E

may be set down as follows:

1. A⊃D (B.C)

2. (BvD) ⊃E

3. DvA /∴E

4. ~E I.P. (Indirect Proof)

5.~(BvD) 2, 4, M.T.

6. ~B.~D 5, De M.

7. ~D.~B 6, Com.

8. ~D 7, Simp.

9. A 3, 8, D.S.

10. B.C 1, 9, M.P.

11. B 10, Simp.

12. ~B 6, Simp.

13. B.~B, 11, 12, Conj.

Unit IV

Quantification: Symbolisation, Proof construction

1. What is quantification theory?

Ans: Quantification, in logic, the attachment of signs of quantity to the predicate or subject of a proposition.

2. How will you symbolise singular proposition in quantification theory?

Ans: In symbolizing singular propositions, we use the small letters 'a' through 'w' to denote individuals, ordinarily using the first letter of an individual's name to denote that individual. Because these symbols denote individuals, we call them 'individual constants'. To designate attributes, we use capital letters, being guided by the same principle in their selection. Thus in the context of the preceding argument, we denote Socrates by the small letter 's' and symbolize the attributes human and mortal by the capital letters 'H' and 'M'. To express a singular proposition in our symbolism, we write the symbol for its predicate term to the left of the symbol for its subject term. Thus we symbolize 'Socrates is human' 'Hs' and 'Socrates is mortal' as 'Ms'.

is humanas he and bic formulations of singular propositions having the same predicate term, we observe them to have a common pattern. The symbolic formulations of the singular propositions 'Aristotle is human', 'Boston is humic forcalifornia is human, Descartes is human', which are 'Ha', Hb. He Halfon each consists of the attribute symbol Hfollowed by an individual constant. We use the expression 'Hx', to symbolize the pattern common to all such singular propositions. The small letter 'x' called an individual variable' is a mere place marker that serves to indicate where an individual constant can be written to produce a singular proposition. The 'Hx' are singular propositions 'Ha', 'Hb', 'Hc', 'd',... are either true or false; but 'H' is neither true nor false, since it is not a proposition. Such expressions as called propositional functions'. These are defined to be expressions that contain individual variables and become propositions when individual con- stants are substituted for their individual variables. Any singular proposi- tion can be regarded as a substitution instance of the propositional function from which it results by the substitution of an individual constant for the individual variable in the propositional function. The process of obtaining a proposition from a propositional function by substituting a constant for a variable is called 'instantiation'. The negative singular propositions 'Aristotle is not human' and 'Boston is not human,' symbolized as '~Ha' and '~Hb', result by instantiation from the propositional function '~Hx', of which they are substitution instances. Thus we see that symbols other than attribute symbols and individual variables can occur in propositional functions.

3. How will you symbolise general proposition in quantification theory?

Ans: General propositions such as 'Everything is mortal' and 'Something is mortal differ from singular propositions in not containing the names of any individuals. However, they also can be regarded as resulting from propositional functions, (not by instantiation, but by the process called 'generalization' or 'quantifica tion') The first example, 'Everything is mortal', can alternatively be ex- pressed as

Given any individual thing whatever, it is mortal.

Here the relative pronoun 'it' refers back to the word 'thing', which precedes it in the statement. We can use the individual variable 'x' in place of the pronoun 'it' and its antecedent to paraphrase the first general proposition as

Given any x, x is mortal.

Then we can use the notation already introduced to rewrite it as

Given any x, Mx.

The phrase 'Given any x' is called a 'universal quantifier', and is symbolized by '(x)'. Using this new symbol, we can completely symbolize our first general proposition as

(x)Mx

We can, similarly, paraphrase the second general proposition, 'Something is mortal', successively as

There is at least one thing that is mortal.

There is at least one thing such that it is mortal.

There is at least one x such that x is mortal.

and as

There is at least one x such that Mx.

The phrase "There is at least one x such that' is called an 'existential quantifier' and is symbolized as '(∃x)'. Using the new symbol, we can completely symbolize our second general proposition as

(∃x)Mx

A general proposition is formed from a propositional function by placing either a universal or an existential quantifier before it. It is obvious that the universal quantification of a propositional function is true if and only if all of its substitution instances are true. Also, the existential quantification of a propo- sitional function is true if and only if it has at least one true substitution instance) If we grant that there is at least one individual, then every proposi- tional function has at least one substitution instance (true or false). Under this assumption, if the universal quantification of a propositional function is true, then its existential quantification must be true as well.

4. Explain the preliminary quantification rules of proving valudity.

Ans: To construct formal proofs of validity for arguments symbolized by means of quantifiers and propositional functions, we must augment our list of Rules of Inference.

1. Universal Instantiation (Preliminary Version). Because the universal quantification of a propositional function is true if and only if all substitution instances of that propositional function are true, we can add to our list of Rules of Inference the principle that any substitution instance of a propositional function can validly be inferred from its universal quantification. We can express this rule symbolically.as

(x)(Φx)

∴ Φν (where v is any individual symbol)

Since this rule permits substitution instances to be inferred from universal quantifications, we refer to it as the 'principle of Universal Instantiation', and abbreviate it as 'UI. The addition of UI permits us to give a formal proof of validity for the argument: 'All humans are mortal; Socrates is human; there- fore, Socrates is mortal'.

1. (x)(Hx ⊃Mx)

2. Hs /∴ Ms

3. Hs ⊃Ms 1, UI

4. Ms 3, 2, M.P.

2. Universal Generalization (Preliminary Version). We can next rule by analogy with fairly standard mathematical practice. A geometer may begin a proof by saying, "Let ABC be any arbitrarily selected triangler Then he may go on to prove that the triangle ABC has some specified attribute Then he may go on to proveles have that attribute. Now, what justifies his final

conclusion? Why does it follow from triangle ABC's having a specified attri bute that all thingles do? The answer is that if no assumption other than its briangular triangle about ABC, then the expression And than denote any triangle whatsoever. And if the argument has established that any triangle whatsoever must have the attribute in question, then it follows that all triangles dhatever must have the notation analogous to that of the geometer in his to any arbitrarily selected triangle'. The hitherto unused small letter referated to denote any arbitrarily selected individual. In this usage, the expression "y is a substitution instance of the propositional function 'x', and it asserts that any arbitrarily selected individual has the property $. Clearly, 'Py follows validly from "(x)(x)" by UI, since what is true of all individuals is true of any arbitrarily selected individual. The inference is equally valid in the other direction, since what is true of any arbitrarily selected individual must be true of all individuals We augment our list of Rules of Inference further by adding the principle that the universal quantification of a propositional function can validly be inferred from its substitution instance with respect to the symbol 'y'. Since this rule permits the inference of general propositions that are universal quantifications, we refer to it as the 'principle of Universal Generalization', and abbreviate it as 'UG'. Our symbolic expression for this second quantification rule is

Φy

∴ (x)(Φx) (where 'y' denotes any arbitrarily

selected individual, and Φy is not

within the scope of any assumption

containing the special symbol 'y')

We can use the new notation and the additional rule UG to construct a formal proof of validity for the argument: 'No mortals are perfect; all humans are mortal; therefore, no humans are perfect'.

1. (x) (Mx⊃~ Px)

2. (x) (Hx ⊃Mx) /∴ (x) (Hx ⊃~Px)

3. Hy ⊃My 2, UI

4. My ⊃~Py 1, UI

5. Hy⊃~Py 3, 4, H.S.

6. (x) (Hx⊃~Px) 5, UG

We can explain the need for the two restrictions in our statement of UG by considering two obviously invalid arguments. First, the silly argument 'Chicago is large, therefore, everything is large'. If we ignored the restriction on UG that the premiss must contain the special symbol 'y', we might be led to construct the following 'proof':

1. Lc/ ∴ (x)Lx

2. (x)Lx 1, UG (wrong)

The mistake occurs at line 2. The individual symbol in line 1 is c, but UG can be used only on a line containing 'y'.

3. Existential, Generalization (Preliminary Version). Because the existential quantification of a propositional function is true if and only if that proposi- tional function has at least one true substitution instance, we can add to our list of Rules of Inference the principle that the existential quantification of a propositional function can validly be inferred from any substitution instance of that propositional function. This rule permits the inference of general propo- sitions that are existentially quantified, so we call it the 'principle of Existential Generalization' and abbreviate it as 'EG'. Its symbolic formulation is

$\Phi\nu$

$\therefore (\exists x)(\Phi x)$ (where v is any individual

symbol)

4.Existential Instantiation (Preliminary Version). One further quantification rule is required., The existential quantification of a propositional function asserts that there exists at least one individual the substitution of whose name for the variable 'x' in that propositional function will yield a true substitution instance of it. Of course, we may not know anything else about that individual. But we can take any individual constant other than 'y', say, 'w', which has had no prior occurrence in that context and use it to denote the individual or one of the individuals whose existence has been asserted by the existential quantification. Knowing that there is such an individual, and having agreed to denote it by 'w', we can infer from the existential quantification of a propositional function the substitution instance of that propositional function with respect to the individual symbol 'to'. We add, as our final quantification rule, the principle that from the existential quantification of a propositional function we may with respect to an valis inter thatnd prior occurrence in that context. The new rule may be written as

$(\exists x)(\Phi x)$

$\therefore \Phi y$ (where is an individual constant,

other than 'y', that has no prior

occurrence in the context)

It is referred to as the 'principle of Existential Instantiation' and abbreviated as EI. We make use of the last two quantification rules in constructing a formal proof of validity for the argument: 'All dogs are carnivorous; some animals are dogs; therefore, some animals are carnivorous'.

1. (x)(Dx ⊃ Cx)

2. (∃x) (Ax.Dx) / ∴ (∃x) (Ax.Cx)

3. Aw. Dw 2, EI

4. Dw ⊃Cu 1, UI

5. Dw .Aw 3, Com.

6. Dw 5, Simp.

7. Cw 4, 6, M.P.

8. Aw 3, Simp.

9. Aw.Cw 8, 7, Conj.

10. (∃ x) (Ax.Cx) 9, EG

We can show the need for the indicated restriction on the use of EI by considering the obviously invalid argument: 'Some cats are animals; some dogs are animals; therefore, some cats are dogs'. If we ignored the restriction on El that the substitution instance inferred by it can contain only an individual constant that had no prior occurrence in the context, we might be led to construct the following 'proof':

1. (∃ x) (Cx.Ax)

2. (∃ x) (Dx.Ax) / ∴ (∃x)(Cx.Dx)

3. Cu.Aw 1, EI

4.Dw.Aw 2, EI (wrong)

5. Cw 3, Simp.

6. Dw 4, Simp.

7. Cw.Dw 5, 6, Conj.

8. (3x)(Cx.Dx) 7, EG

The mistake here occurs at line 4. The second premiss assures us that there is at least one thing that is both a dog and an animal. We are not free to use the symbol 'w' to denote that thing, however, because 'w' has already been used to denote one of the things that was asserted by the first premiss to be both a cat and an animal. To avoid errors of this sort, we must obey the indicated restriction in using EI. It should be clear that whenever we use both El and UI in a proof to instantiate with respect to the same individual constant, we must use El first) (It has been suggested that we could avoid the necessity of using El before UI by changing the restriction on El to read 'where is an individual constant, other than 'y', that was not introduced into the context by any previous use of EI.' Even apart from its apparent circularity, that formulation would not prevent the construction of an erroneous 'formal proof of validity' for such an invalid argument as 'Some men are handsome. Socrates is a man. Therefore, Socrates is handsome.')

Like the first nine Rules of Inference presented in Section 3.1, the four quantification rules UI, UG, EG, and EI can be applied only to whole lines in a proof.

Unit V

Probability: Theories of addition, multiplication and their joint application
Mill's method of experimental enquiry

1. What is probability?

Ans : A is defined as the ratio of two numbers a and b is, P(A) = a/b, where a is the number of ways which are favorable to the occurrence of A and b is the total of the number of outcomes of the event.

In simple terms, probability refers to the chance or likelihood of a particular event happening.

Terminologies

Event: An event is an outcome of an experiment.

Experiment: An experiment is a process which is performed to understand and observe all possible outcomes.

Sample set: A sample set is a set of all outcomes of an experiment.

2. What is theory of addition in probability?

Ans: Addition Rules for computing probability

Addition rule for mutually exclusive events:

$P(A \cup B) = P(A) + P(B)$.

Addition rule for not mutually exclusive events:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Probability by Boolean way

Suppose there are two events **A** and **B** which are not mutually exclusive then, the addition rule is,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$. ——— (1)

Now, by using Boolean Theory, it can be proved that both the equations are the same.

The event A is counted as the addition of the event A taking place when the event B is present ***[A B]*** and when event B is not present ***[A (~B)]*** which is shown below:

A = A B + A (~B)

∴ A (~B) = A - A B

Similarly,

B = B A + B (~A)

∴ B (~A) = B - B A

Now,

A or B = A B + A (~B) + B (~A)

Put the values of A (~B) and B (~A) respectively in the above equation,

A or B = A B + A - A B + B - B A, where A B gets cancelled out,

therefore, the remaining part is A or B = A + B - A B

∴ P(A or B) = P(A) + P(B) - P(A and B)

∴ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ which is equal to the first equation.

Examples

From a pack of well-shuffled cards, a card is drawn at random, then what is the probability that the selected card is a king or a diamond?

King (K) or Diamond (D) = K D + K (~D) + D (~K) = 1 + 3 + 12 = 16

Probability = Favorable instances / Total instances = 16 / 52

Note:

If for instance, for a king in a deck of cards, then there are two approaches you can take into consideration.

1. The frequency approach is where K + (~K) = 52 (because the total cards are 52).

2. The proportional approach is where K + (~K) = 1 or 100 %.

The employees of a company were surveyed on their educational background (whether they have any degree or not) and marital status (either single or married).

Out of 800 employees, 500 has college degrees, 200 were single, and 150 were single college graduates. The probability that an employee of the company is single or has a college degree is:

	D	(~D)
S	SD	S(~D)
(~S)	(~S)D	(~S)(~D)

```
Where,  S - Single,  D - College Degree
```

A boolean table to describe four different conditions.

Here, S refers to an employee being single, whereas (~S) means married. Likewise, D refers to having a college degree, while (~D) means not a college graduate.

Now, when we enter the values from the question, we get the following table.

	D	(~D)	Total
S	150	50	200
(~S)	350	250	600
Total	500	300	800

Explanation of the above table

To begin with, we had ***500*** college graduates, so total D would be 500. Out of which ***150*** were single, so [S D] cell is filled up with 150. so, [(~S) D] cell would automatically be 500 - 150 = ***350***. Therefore, the first column is filled up.

Now, there were 200 single employees, so total S would be 200. Out of total 200 single employees, 150 were having a college degree, so the number of single employees who are not college graduates [S (~D)] cell would be 200 - 150 = ***50***. At last, the total number of employees who are neither single nor having any college degrees are 800 - [150 + 50 + 350] = ***250***.

The question is to find the probability of employees who are either single or have a college degree,

S or D = S D + (~S) D + S (~D) = 150 + 350 + 50 = 550

Probability (S ∪ D) = Favorable instances / Total instances = 550 / 800 = 11 / 16

Thus, this is how you can calculate any probability problems by creating a 2 x 2 table and then divide **favorable events** over **total events**.

3. What is theory of addition in probability?

Ans: **Multiplication Rule**

A statistical property of probability theory is used to find out the probability of happening events in more than one task. Suppose two dice are rolling followed by one after another and find out probability of rolling 3 on one dice and rolling odd number in second dice. Here two tasks are happening, one is rolling dice first time and second one is rolling dice second time simultaneously and we have to find out probability of happening events. These types of problem can be solved by using multiplication probability theorem.

“Multiplication probability theorem is applied to find out compound probability of two and more events of more than one task.”

In other words, Multiplication probability theorem is used to find probability of events occurring in different task. Intersection concept is used to find probability of all occurring events. Intersection concept is also a key concept of mathematical probability and set theory similar as union concept. It is denoted as ∩.

Since, two events can occur simultaneously in different two tasks. If an event depends on the event of different trial, it is dependent event otherwise it will be independent event. Mathematical formulation for independent and dependent will be slightly different. It is known as multiplication probability formulation for dependent event and multiplication probability formulation for independent event respectively.

Let us discuss multiplication probability formulation for dependent and independent event.

Mathematical Formulation

Multiplication probability formulation for dependent event is slightly different from independent events happening in more than two tasks. Multiplication probability formulation for dependent event is generalization of the multiplication probability formulation for independent event.

Multiplication probability formulation for dependent event

Suppose E and F are two dependent events occurring in two different tasks. Since E and F are two dependent event i.e. outcome of one is affecting the outcome of second event.

One more thing is that second task will happen after completed first task. So that, conditional probability used in multiplication rule.

Multiplication probability formulation for two dependent events is given as:

$?(? \cap ?) = ?(?|?)?(?).$

This formulation can be read as:

Probability of events E and F = probability of event E given F event has already occured × probability of event F.

Multiplication probability formulation for three dependent event E, F and G is given as:

$$?(? \cap ? \cap ?) = ?(?)?(?|?)?(?|? \cap ?).$$

This formulation can be read as:

Probability of event E, F and G = probability of event E × probability of event F given event E × probability of event G given events E and F.

Similarly, multiplication probability formulation can be extended to n dependent events.

4. What is meant by Mill’s method of experimental enquiry?

Ans: With his methods of experimental inquiry, it was J. S. Mill’s (1806–1873) aim to developmeans of induction that would promote a search for causes (Flew, 1984). Mill recognized induction as a process whereby one generalizes from experience but it was his belief, beyond that, that all induction involves a search for causes, and that his methods were intended to support this (Day, 1964). Furthermore, the methods, he thought, would contribute to a definition of “cause.” To Mill, causal law meant “uniformity of succession” which, presumably, is consistent with Hume’s constant conjunction, and refers to events that are invariable antecedents, i.e., when the antecedent occurs it is always followed by the consequent. Mill went further, however, in proposing that causal laws are proved upon the basis of the law of universal causation, the idea that each event has its cause (Newton's determinism). This would be bolstered when supported by experimental methods.

5. What do you mean by The Method of Agreement?

   Ans: According to the method of agreement, if two or more examples of a phenomenon only share in a single antecedent condition, that single condition is the cause of the examples of the phenomena (Hung, 1997). By such reasoning, if, in all cases of tree rot, I identify a type of tick to be present, I conclude the tick to have caused the tree rot. The tick, however, may be ubiquitous and found in trees that are not rotting; there could be an unobservable virus. As Hung points out, the method is open to difficulties in interpretation. There may be a relationship that is merely coincidental between the antecedent and the consequent, as was the case in the preceding example. Second, the cause may not have been included in the antecedent conditions, e.g., the miniscule virus. Furthermore, there could be multiple causes, such as the presence of the virus and a certain temperature range. On top of that, the cause could be non-uniform; the same end result may be caused by a number of factors acting separately. With tree rot, the rot may be the same but one tree may rot due to soil that is too damp, another because of woodpeckers, and another because of insect infestation, with our ubiquitous tick still present in every instance. Finally, it is possible that two events may occur in succession, invariably, without a causal relation between them, such as with thunder and lightning, both of which share in a cause but differ in the time their effects take to reach us. We might assume the first experience (seeing lightning) to be the cause of the second experience (hearing thunder). The differences in rates of wave travel produce the temporal differences in receipt of stimulation at the receptors. One has to conclude that the above problems make the generalization questionable.

6. What do you mean by The Method of Difference?

Ans: With the method of difference, there are cases where the phenomenon under scrutiny is sometimes present and sometimes absent, and in which all other elements, but one, remain constant (over those instances of the "phenomenon-present" and the "phenomenon-absent"). Given those conditions, the element that is present when the other is present, and absent when the other is absent, is causally related to the other. In this case, given that all other aspects within the circumstances being considered have to be the same, we have an example of what, in modern terminology, is called the control of confounding variables.

7. What do you mean by Joint Method of Agreement and Difference?

Ans: With the joint method of agreement and difference, one gathers a number of instances of positive and negative cases (without everything the same being constant, as with the method of difference above). The multiplication of the number of instances observed is intended to make the method more reliable. The approach expands upon the method of agreement since both positive and negative instances are drawn upon rather than just positive instances. Through the use of this method one concludes that if, due to a process of elimination, the antecedent condition is always present when the consequent is present, but is never present when the consequent is not present, that the antecedent condition is the cause.

8. What do you mean by Method of Concomitant Variation?

Ans: With the method of concomitant variation one is looking for circumstances where one of the elements, say the antecedent, is varying in magnitude and cases where some other variable varies in a similar manner (Day, 1964). Under such circumstances one determines the first to be causally related to the second if an increase (or decrease) is accompanied by an increase (or a decrease) in the second. For instance, the amount of time that the sun is present during the day is associated with the average temperature. When the sun is briefly present temperature is low and vice versa. (This is what we would now refer to as a positive correlation.)

9. What do you mean by Method of Residues?

Ans: With the method of residues, one commences with the knowledge that something (A), the antecedent, causes an effect (E), and, furthermore, that the antecedent contains within it an element (A:1) that also has a known effect (E:1). Now, given these conditions, it can be concluded that the first cause (A) minus its component cause (A:1) will equal the first effect (E) less the second, componential, effect (E:1) or (A – A:1 = E – E:1). A person weighing her or himself, for instance, who is rather shy around doctors, can get on a scale fully clothed and then go behind a screen and remove all clothing. The weight of the clothing subtracted from the weight of the clothed person will give the person's weight unclothed. The method, as Hung (1997) pointed out, has its own problems since some causes may not be additive. For instance, water is composed of both hydrogen and oxygen. Both are gases that are flammable but, added together, they cause a dousing of flame; conversely, water douses fire, but, if it has one of its components removed, it will result in unwanted consequences when applied to flame.