



**2024/FYUG/EVEN/SEM/
MATDSC-152T/127**

FYUG Even Semester Exam., 2024

MATHEMATICS

(2nd Semester)

Course No. : MATDSC-152T

(Integral Calculus and Vectors)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

SECTION—A

Answer any ten of the following questions :

2×10=20

1. Express $\int_a^b f(x) dx$ as the limit of sum.

2. Evaluate :

$$\int_{-5}^5 |x+2| dx$$



(2)

3. Prove that

$$\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$$

4. If $I_n = \int x^n \cos ax dx$ and $J_n = \int x^n \sin ax dx$, then show that $aI_n = x^n \sin ax - nJ_{n-1}$.

5. Evaluate :

$$\int_0^{\pi/2} \cos^6 x dx$$

6. Evaluate :

$$\int_0^1 x^2 (1-x)^{3/2} dx$$

7. Write the formula to find the length of the curve in Cartesian form and polar form.

8. The circle $x^2 + y^2 = a^2$ revolves around the axis. Write down the surface area and the volume of the whole surface generated.

9. Find the volume generated by revolving about OX, the area bounded by $y = x^3$ between $x = 0$ and $x = 2$.

10. Prove that

$$\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$$

(3)

11. Find the value of λ if the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $\lambda\hat{i} - 4\hat{j} + 5\hat{k}$ are coplanar.

12. Find the vector equation of a plane in normal form.

13. Show that

$$\frac{d}{dt} \left(\vec{a} \times \frac{d\vec{b}}{dt} - \frac{d\vec{a}}{dt} \times \vec{b} \right) = \vec{a} \times \frac{d^2\vec{b}}{dt^2} - \frac{d^2\vec{a}}{dt^2} \times \vec{b}$$

14. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, then show that

$$\vec{\nabla} r^n = nr^{n-2} \vec{r}$$

15. Show that the vector

$$\vec{V} = (4xy - z^3)\hat{i} + 2x^2\hat{j} - 3zx^2\hat{k}$$

is irrotational.

SECTION—B

Answer any five of the following questions :

10×5=50

16. (a) Evaluate :

5

$$\lim_{n \rightarrow \infty} \left[\frac{1^2}{n^3 + 1^3} + \frac{2^2}{n^3 + 2^3} + \dots + \frac{n^2}{2n^3} \right]$$



(b) Prove that

$$\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2} \log 2 \quad 5$$

17. (a) Evaluate $\int_0^2 e^x$ as the limit of a sum. 5

(b) Evaluate : 5

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}} = \frac{\pi}{2}$$

18. (a) If $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$, then show that

$$I_n = \frac{1}{n-1} - I_{n-2}$$

Hence find the value of $\int_0^{\pi/4} \tan^6 \theta \, d\theta$.

$$3+2=5$$

(b) Obtain the reduction formula for

$$\int \sec^n x \, dx \quad 5$$

19. (a) If $I_n = \int_0^{\pi/2} x^n \sin x \, dx$ and $n > 1$, then show that

$$I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1} \quad 5$$

(b) Obtain the reduction formula for

$$\int \sin^m x \cos^n x \, dx \quad 5$$

20. (a) Find the length of the perimeter of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$. 5

(b) Find the surface area of the solid generated by revolving the cardioid $r = a(1 - \cos \theta)$ about the initial line. 5

21. (a) Prove that the length of one arc of the cycloid $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$ is $8a$. 5

(b) Find the volume generated by the revolution about x-axis of the area bounded by the loop of the curve $y^2 = x^2(2-x)$. 5

22. (a) (i) If $\vec{\alpha} = 2\hat{i} + 3\hat{j} - 5\hat{k}$, $\vec{\beta} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{\gamma} = 4\hat{i} + 2\hat{j} + 6\hat{k}$, then find $(\vec{\alpha} \times \vec{\beta}) \cdot \vec{\gamma}$. 2

(ii) Show that

$$(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a}) = [\vec{a} \, \vec{b} \, \vec{c}] \vec{c} \quad 3$$

(b) (i) Find the vector equation of the line joining the points $\hat{i} - 2\hat{j} + \hat{k}$ and $3\hat{k} - 2\hat{j}$. 2



(6)

- (ii) Find the vector equation of the sphere with the join of two given points as diameter.

3

23. (a) Show that the equation of the straight line passing through $A(\vec{a})$ and parallel to the unit vector $\hat{e}$ is $(\vec{r} - \vec{a}) \times \hat{e} = 0$. Also deduce that the equation of the straight line passing through the origin is $\vec{r} \times \hat{e} = 0$.

3+2=5

- (b) Find the condition that the two spheres $\vec{r}^2 - \vec{r} \cdot \vec{c} + k = 0$ and $\vec{r}^2 - 2\vec{r} \cdot \vec{c}' + k' = 0$ may intersect orthogonally.

24. (a) If $\vec{u}(t)$ and $\vec{v}(t)$ be two differentiable functions of the scalar t , then show that

$$\frac{d}{dt}(\vec{u} \times \vec{v}) = \vec{u} \times \frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt} \times \vec{v}$$

$$\text{Hence show that } \frac{d}{dt}\left(\vec{u} \times \frac{d\vec{u}}{dt}\right) = \vec{u} \times \frac{d^2\vec{u}}{dt^2}.$$

3+2=5

- (b) If $\vec{f}$ and $\vec{g}$ be two vector point functions, then prove that

$$\begin{aligned} \text{grad}(\vec{f} \cdot \vec{g}) &= \vec{f} \times \text{curl } \vec{g} + \vec{g} \times \text{curl } \vec{f} \\ &+ (\vec{f} \cdot \nabla)\vec{g} + (\vec{g} \cdot \nabla)\vec{f} \end{aligned}$$

5

(7)

25. (a) (i) Show that

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{a}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{a}) - \vec{\nabla}^2 \vec{a}$$

3

- (ii) Prove that $\text{curl}(\text{grad } \vec{f}) = \vec{0}$.

2

- (b) Show that a necessary and sufficient condition for a vector $\vec{r} = \vec{f}(t)$ to have constant magnitude is

$$\vec{f} \cdot \frac{d\vec{f}}{dt} = 0$$

5
