



**2023/FYUG/ODD/SEM/
MATDSC-101T/140**

FYUG Odd Semester Exam., 2023

(Held in 2024)

MATHEMATICS

(1st Semester)

Course No. : MATDSC-101T

(Higher Algebra and Trigonometry)

Full Marks : 70

Pass Marks : 28

Time : 3 hours



*The figures in the margin indicate full marks
for the questions*

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SECTION—A

Answer *ten* questions, taking *two* from each Unit :

2×10=20

UNIT—I

- 1. Find the general value of θ which satisfies the following equation :**

$$(\cos \theta + i \sin \theta)(\cos 2\theta + i \sin 2\theta) \dots$$

$$(\cos n\theta + i \sin n\theta) = 1$$

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(Turn Over)

(2)

2. Find all the values of $(-1)^{1/3}$.
3. Expand $\sin^3 x$ in ascending powers of x .

UNIT—II

4. Prove that $i^i = e^{-(4n+1)\pi/2}$.

5. Show that

$$\pi = 2\sqrt{3} \left\{ 1 - \frac{1}{3 \cdot 3} + \frac{1}{5 \cdot 3^2} - \frac{1}{7 \cdot 3^3} + \dots \right\}$$

6. Express $\sin(x+iy)$ in the form of $A+iB$.



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UNIT—III
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7. The relation R on $\mathbb{Z}$ defined by $(a, b) \in R$ iff $|a-b|=2023$. Show that R is symmetric but not transitive.
8. Let R be an equivalence relation on A . Show that for any $a, b \in A$, $[a] = [b]$ iff $a \in [b]$, where the symbol $[]$ represents equivalence class.
9. Write the negation of the statement $\forall \alpha \in A, \exists x \in B$ such that $x > \alpha$.

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UNIT—IV

10. If α , β and γ are the roots of the equation $x^3 - 3x^2 + 2x - 7 = 0$, then find the value of $\alpha\beta + \beta\gamma + \gamma\alpha$.
11. Find the equation whose roots are reciprocal of the roots of $x^3 - 6x^2 + 11x - 6 = 0$.
12. If $x + y + z = 1$, then prove that

$$(1-x)(1-y)(1-z) > 8xyz$$

UNIT—V

13. What do you mean by canonical form of matrices?
14. Define rank of a matrix.
15. Show that the set $\{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ is LI.

SECTION—B

Answer *five* questions, taking *one* from each Unit :

$$10 \times 5 = 50$$

UNIT—I

16. (a) State de Moivre's theorem and prove it for positive integral index.

4

(4)

(b) (i) If $(1+x)^n = a_0 + a_1x + a_2x^2 + \dots$, then prove that

$$a_0 - a_2 + a_4 - \dots = 2^{n/2} \cos^{n/4} \pi \quad 3$$

(ii) Expand $\cos 7\theta$ in ascending powers of $\cos \theta$. 3

17. (a) (i) If $x = \cos \theta + i \sin \theta$ and $1 + \sqrt{1-a^2} = na$, then prove that

$$1 + a \cos \theta = \frac{a}{2n} (1 + nx) \left(1 + \frac{n}{x} \right) \quad 3$$

(ii) Expand $\sin x$ in ascending powers of x . 3

(b) Prove that

$$\frac{\sin^3 \theta}{3} - \frac{\theta^3}{3} + \frac{\theta^5}{5} - \frac{\theta^7}{7} + \dots = \frac{\theta^3}{3} (1 + 3^2 + 3^4 + \dots) \quad 4$$

UNIT—II

18. (a) State and prove Gregory's series. 4

(b) (i) If $\cos^{-1}(\alpha + i\beta) = x + iy$, then show that $\alpha^2 \sec^2 y + \beta^2 \operatorname{cosec}^2 y = 1$. 3

(ii) Find the sum of the series

$$\cos \theta - \frac{1}{2} \cos 2\theta + \frac{1}{3} \cos 3\theta - \frac{1}{4} \cos 4\theta + \dots \quad 3$$

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19. (a) (i) Prove that

$$\log(x + iy) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \frac{y}{x} \quad 3$$

(ii) If θ lies between 0 and $\pi/2$, then prove that

$$\tan^{-1} \left(\frac{1 - \cos \theta}{1 + \cos \theta} \right) = \tan^2 \frac{\theta}{2} - \frac{1}{3} \tan^6 \frac{\theta}{2} + \frac{1}{5} \tan^{10} \frac{\theta}{2} - \dots \quad 3$$

(b) Find the sum of the series

$$\cos \theta + \frac{\operatorname{cosec} \theta}{1} \cos 2\theta + \frac{\operatorname{cosec}^2 \theta}{2} \cos 3\theta + \dots \quad 4$$

UNIT—III

20. (a) State that the relation of 'congruence modulo n ' is an equivalence relation on $\mathbb{Z}$. 5

(b) Show that—

(i) $(p \wedge q) \Rightarrow (p \vee q)$ is a tautology;

(ii) $(\neg p \wedge q) \wedge (p \vee (\neg q))$ is a contradiction. 2+3=5

21. (a) Show that a partition of a non-empty set induces an equivalence relation on A such that the equivalence classes are precisely the members of A . 5

(6)

- (b) (i) Write the following statement using quantifiers and other symbols as required :

For every positive real number ϵ , there exists a natural number n_0 such that the reciprocal of n_0 is less than ϵ .

2

- (ii) Write the following statement as an implication :

If x is greater than 2, then x^2 is greater than 4.

Also, write its converse and contra-positive.

3

UNIT—IV

22. (a) (i) The sum of two roots of the equation $x^3 + a_1x^2 + a_2x + a_3 = 0$ is zero. Show that $a_1a_2 = a_3$.

3

- (ii) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the equation whose roots are $\frac{1}{\alpha^2}, \frac{1}{\beta^2}$ and $\frac{1}{\gamma^2}$.

3

- (b) State and prove Cauchy-Schwarz inequality.

4

(7)

23. (a) (i) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of $\Sigma \alpha^2 \beta$. 3
- (ii) Find the nature of the roots of the equation $x^3 + x^2 - 16x + 20 = 0$. 3
- (b) Solve $x^3 - 30x + 133 = 0$ by Cardan's method. 4

UNIT—V

24. (a) Show that the rank of the transpose of a matrix is the same as that of the original matrix. 5
- (b) Solve by Gaussian elimination method : 5

$$\begin{aligned} x + y + z &= 6 \\ 2x - y + 2z &= 6 \\ 2x + 2y + z &= 9 \end{aligned}$$

25. (a) Find the rank of the matrix

$$\begin{pmatrix} 2 & 2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ -1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{pmatrix}$$

by reducing it to normal form. 5



(8)

(b) (i) Prove that every singleton set containing non-zero vector is LI. 2

(ii) Show that the vectors $(1, 1, 0)$, $(1, 3, 5)$ and $(2, 2, 0)$ in $\mathbb{R}^3$ are LD. 3



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